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THE CONFORMAL MAPPING OF THE INTERIOR OF THE UNIT CIRCLE ONTO THE INTERIOR OF A CLASS OF SMOOTH CURVES

by Thomas F. Moriarty and Will J. Worley

Prepared by

UNIVERSITY OF ILLINOIS

Urbana, Ill.

for

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SUMMARY

The coefficients of the series representing the conformal mapping of the interior of the unit circle onto the interior of $\left|\frac{X}{A}\right|^{\alpha} + \left|\frac{Y}{B}\right|^{\beta} = 1$ are tabulated for various values of B/A, α and β.

Precise graphical comparisons of several cases have been included to graphically illustrate the nature and accuracy of the conformal mappings.

A computer program (Fortran IV computer language) has been listed to aid those interested in cases not included in the report.

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INTRODUCTION

1. Objective of General Project and the Relation of Current Report

The primary objective of the current research has been the optimun design of structural shapes. The optimization was to be restricted only to the relatively large class of two and three dimensional shapes defined by the equations

$$\left|\frac{X}{A}\right|^{\alpha} + \left|\frac{Y}{B}\right|^{\beta} = 1 \qquad (1)$$

$$\left|\frac{X}{A}\right|^{\alpha} + \left|\frac{Y}{B}\right|^{\beta} + \left|\frac{Z}{C}\right|^{\gamma} = 1 \qquad (2)$$

where A, B, C, α, β and γ are positive constants which need not be integers. Further the three dimensional shape resulting from revolving the curve defined by (1) about the x-axis was to be included.

In order to provide material useful to the space program, it was decided to initiate the research with a determination of the geometrical and inertial properties of the above classes of shells [1,2].* This was followed with a study of shells of revolution which were optimized with respect to maximum enclosed volume and minimum weight [3]. A study on the vibration of beams was also reported [4] in which the beam cross-section was defined by (1).

Since bulkheads for bodies of type (2) require plate shapes of type (1) investigation was continued on clamped plates defined by (1). Previous solutions for the load-deflection behavior of clamped plates have been confined to the circle, ellipse, square, rectangle and skew slab or diamond shape. In order to insure the accuracy of the determined deflection of plates with shapes defined by (1) that were not a circle, ellipse, square or rectangle, it was decided to apply three different methods to determine the deflections. These were: the conformal mapping method, the method of collocation and the energy method.

*Numbers in brackets refer to the references.

In investigating the conformal mapping approach, it was noted that there was a severe gap between the theoretical results concerning the construction of conformal maps and the actual availability of those conformal maps. This report contains a detailed discussion of the construction of the conformal maps and a listing of the conformal maps needed in the conformal mapping approached for the determination of the deflection of plates.

Since conformal maps are used in a wide variety of problems besides the determination of the deflection of plates, it was decided to publish this work separately so that its general nature could be more easily recognized and exploited.

2. Purpose of Report

Many two-dimensional physical problems have been solved with the aid of complex variables. These treatments frequently assume the availability of an appropriate conformal mapping. See for example Muskhelishvili [5], Mansfield [6] and Sokolnikoff [7]. Often appropriate conformal mappings are not available.

The purpose of this report is to make available the conformal mappings of the interior of the unit circle onto the interior of a relatively general class of curves which includes as special cases ellipses as well as curves which are essentially rectangles.

Those who have had any encounter with digital computers soon learn to be wary of proposed solutions, no matter how plausible. Until every single part of a numerical solution of a problem is carefully checked and tested, one cannot be sure that considerations of accuracy versus computer running time or computer storage will permit a complete solution.

The basic mathematical theory for the determination of conformal mappings is treated first, then the theory is specialized for the curves of interest in this report. Next a complete listing of the actual computer program is presented for those who may wish mappings which are not tabulated. In addition to a table of accurate conformal mappings graphs of some of these mappings are included to indicate relative accuracy.

Comments contained in the listing of the program (see in particular the note at head of subroutine GETA) should prove helpful to those interested in the limitations of the method used to obtain the conformal maps when shapes severely different from a circle are considered.

3. Acknowledgment

This project was sponsored by the National Aeronautics and Space Administration, Office of Advance Research and Technology, Applied Mathematics Branch, of which Dr. Raymond H. Wilson is Chief.

This investigation was part of the work of the Engineering Experiment Station of which Professor Ross J. Martin is Director and was conducted in the Department of Theoretical and Applied Mechanics of which Professor Thomas J. Dolan is Head.

Both the IBM 7094 and the System/360 computer facilities were used. This equipment is operated by the Department of Computer Science, University of Illinois with support from National Science Foundation.

The authors wish to thank Mr. Tom E. Breuer and Mr. Ekkehard F. Wiechering who wrote and/or checked out various subroutines in the program. The authors also wish to thank Miss Marjorie A. Moretz who performed some of the initial spade work for the project.

DERIVATION OF INTEGRAL EQUATION FOR DETERMINING CONFORMAL TRANSFORMATION OF INTERIOR REGIONS

The determination of a suitable transformation for conformally mapping the interior of the unit circle onto a simple connected region R bounded by curve C shown in Fig. 1 has been shown [8] to reduce to the solution of a Fredholm Integral Equation of the second kind.

Any simply connected region R can be made to contain the origin of the z-plane and intersect the x-axis at 1. Let us map the origin of the z-plane onto the origin of the ζ or image plane and (1,0) of the z-plane onto (1,0) of the image plane. Let $\zeta = \rho e^{i\theta}$. Then consider

$$F(z) = \ln\zeta = \ln\rho + i\theta \tag{1}$$

By Cauchy's theorem

$$F(z) = \frac{1}{2\pi i}\int_C \frac{F(t)}{t-z} - \frac{1}{2\pi i}\int_{C_\varepsilon} \frac{F(t)\,dt}{t-z} + \frac{1}{2\pi i}\int_{2-1} \frac{F(t)\,dt}{t-z} \tag{2}$$

where C_ε is the circle around the origin with radius ε and 2-1 is the cut from (1,0) to the circle C_ε as shown in Fig. 2.

$$\lim_{\varepsilon \to 0} \frac{1}{2\pi i}\int_{C_\varepsilon} \frac{F(t)\,dt}{t-z} = 0$$

Also

$$\lim_{\varepsilon \to 0} \frac{1}{2\pi i}\int_{2-1} \frac{F(t)\,dt}{t-z} = -\ln\left(\frac{z-1}{z}\right)$$

since $F(z)$ on side 1 of cut equals $F(z) - 2\pi i$ on side 2 of cut. On curve C $\rho = 1$ therefore from (1) $F(z) = i\theta$. Now (2) reduces to

$$F(z) = \frac{1}{2\pi}\int_C \frac{\theta\,dt}{t-z} - \ln\left(\frac{z-1}{z}\right) \tag{3}$$

Introducing the changes in variable

$$t - z = r\,e^{i\phi}$$

$$
\begin{aligned}
-z &= r_0 e^{i\phi_0} \\
1 - z &= r_1\, e^{i\phi_1} \\
\beta &= \phi_1 - \phi_0 \\
dt &= e^{i\alpha}\, du
\end{aligned}
$$

(3) becomes

$$F\,(z) = \frac{1}{2\pi} \int_C \frac{\theta e^{-i\phi}\, e^{i\alpha}}{r}\, du - \ln\,(\frac{r_1}{r_0}) - \beta i \tag{4}$$

See Fig. 3 for geometrical significance of the new variables. From Fig. 3 one obtains

$$\alpha - \phi = \frac{\pi}{2} - (n_t, r)\ .$$

Where (n_t, r) indicates the angle between the inward normal to C and r. Substituting in this relationship and utilizing the Euler formula (4) becomes

$$F\,(z) = \frac{i}{2\pi} \int_C \frac{\theta\ [\cos\,(n_t, r) - i\,\sin\,(n_t, r)]}{r}\, du - \ln\,(\frac{r_1}{r_0}) - \beta i \tag{5}$$

Comparing (1) with (5) and equating real and imaginary parts.

$$\ln\rho = \frac{1}{2\pi} \int_C \frac{\theta\,\sin\,(n_t, r)}{r}\, du - \ln\,(\frac{r_1}{r_0})$$

$$\theta = \frac{1}{2\pi} \int_C \frac{\theta\,\cos\,(n_t, r)}{r}\, du - \beta\ . \tag{6}$$

As z approaches the boundary C, as shown in Fig. 4, (6) assumes the form of a Fredholm Integral Equation of the second kind

$$\theta\,(s) = \frac{1}{\pi} \int_C \frac{\theta\,(u)\,\cos\,(n_t, r)}{r}\, du - 2\beta\,(s) \tag{7}$$

SPECIALIZATION OF FREDHOLM INTEGRAL EQUATION FOR REGIONS OF THE SHAPE $\left|\frac{X}{A}\right|^{\alpha} + \left|\frac{Y}{B}\right|^{\beta} = 1$

The kernel, K, of (7) may be written with variables that are more readily obtainable geometrically. From (7)

$$K = \frac{1}{\pi} \frac{\cos (n_t, r)}{r} \tag{8}$$

For the first quadrant

$$K_a = \frac{1}{\pi} \frac{(x_i - x_n)\cos \phi_i + (y_i - y_n)\sin \phi_i}{(x_i - x_n)^2 + (y_i - y_n)^2} \quad , \; n \neq i \tag{9}$$

where (x_n, y_n) is a point on the curve in the first quadrant,

(x_i, y_i) is another distinct point on the curve in the first quadrant,

ϕ_i is the angle the outward normal on the curve at (x_i, y_i) makes with the positive x axis. By suitable reflections about the x and y axes the kernels for the second, third and fourth quadrants are obtained as

$$K_b = \frac{1}{\pi} \frac{(x_i + x_n)\cos \phi_i + (y_i - y_n)\sin \phi_i}{(x_i + x_n)^2 + (y_i - y_n)^2} \tag{10}$$

$$K_c = \frac{1}{\pi} \frac{(x_i + x_n)\cos \phi_i + (y_i + y_n)\sin \phi_i}{(x_i + x_n)^2 + (y_i - y_n)^2} \tag{11}$$

$$K_d = \frac{1}{\pi} \frac{(x_i - x_n)\cos \phi_i + (y_i + y_n)\sin \phi_i}{(x_i - x_n)^2 + (y_i + y_n)^2} \tag{12}$$

respectively.*

---

*For work concerned with curves comprised of circular arcs and straight line segments having various degrees of symmetry see Wilson [9], Rohm & Haas Company [10], and Wilson and Becker [11].

For the case $n = i$ it can be shown [12] p. 95 that

$$K_a = \frac{1}{\pi} \frac{\text{curvature at } (x_n, y_n)}{2}$$

The curvature is considered negative if the center of curvature lies on the same side of the curve as the outward normal.

One now writes (7) in the form

$$\theta(s) = \int_{C_a} \theta(u)_a K_a du + \int_{C_b} \theta(u)_b K_b du + \int_{C_c} \theta(u)_c K_c du + \int_{C_d} \theta(u)_d K_d du - 2\beta(s) \tag{13}$$

where C_a, C_b, C_c, C_d refer to that portion of the boundary, C, in the respective quadrants.

Using Gaussian quadrature, each of the four integrals is of the form

$$\int_o^L f(u)du = \frac{L}{2} \int_{-1}^{1} f(\frac{L}{2} u' + \frac{L}{2}) du' \tag{14}$$

Where L is the length of the curve C_a. Since the entire curve has double symmetry, L also represents the length of C_b, C_c, and C_d.

Let $S_1' < S_2' \cdot \cdot \cdot \cdot \cdot < S_i'$ be the abscissas for Gaussian quadrature in the interval (1, -1). For Gaussian quadrature the abscissas for the interval (0,L) will be

$$u_1 = \frac{L}{2} S_1' + \frac{L}{2}$$

$$u_2 = \frac{L}{2} S_2' + \frac{L}{2}$$

$$\vdots$$

$$u_i = \frac{L}{2} S' + \frac{L}{2}$$

Now specify those points (x_i, y_i) appearing in Fig. 4 to be points for which

$$u(x_i, y_i) = u_i$$

with (x_n,y_n) being a specific (x_i,y_i).

It should be noted that these (x_i,y_i) are in the first quadrant with the corresponding points in the other three quadrants being obtained by suitable reflections about the x and y axes. Thus, it is possible to define K_b, K_c, K_d in terms of (x_i,y_i) of the first quadrant.

Recall that θ determines a point on the curve which has been mapped onto the unit circle. Due to the double symmetry of the curve, C, one has

$$\begin{aligned} \theta_a &= \theta_i \\ \theta_b &= \pi - \theta_i \\ \theta_c &= \pi + \theta_i \\ \theta_d &= 2\pi - \theta_i \end{aligned} \tag{16}$$

Let W_i be the Gaussian weights associated with u_i. Angle β, Fig. 4, can be expressed as

$$\beta_n = \pi - \tan^{-1}\left(\frac{y_n}{x_n}\right) - \tan^{-1}\left(\frac{y_n}{1-x_n}\right) \tag{17}$$

Now writing the integrals of (13) as summations using Gaussian quadrature and making use of (14) and (16), one has for (13)

$$\begin{aligned} &\theta_n - \frac{L}{2}\sum_{i=1}^{I} \theta_i W_i \,[K_a - K_d + K_c - K_b] \\ &= \frac{L\pi}{2}\sum_{i=1}^{I} W_i \,[K_c + K_b] + L\,\pi \sum_{i=1}^{I} W_i K_d - 2\beta_n \end{aligned} \tag{18}$$

where I is the number of points (x_i,y_i) and β_n is defined by (17). As n takes on all values $n = 1, \ldots I$, we see that (18) forms a system of I linear algebraic equations involving the I unknowns, θ_i. On solving this system of equations a table, T, of (x_i,y_i) versus θ_i is generated.

The entries θ_i in this table will be unevenly spaced due to the use of Gaussian quadrature and hence will be unsuitable for the Fourier analysis that is to follow. If sufficient care was taken in setting up and solving the system of simultaneous linear equations the table generated will be highly accurate and therefore will be a suitable base on which to construct a table with more evenly spaced entries for the same range by a suitable nth degree Lagrange interpolation. This new table will be denoted T'.

It is desired, ultimately to obtain the conformal transformation in the form

$$z = x + iy = \sum_{n=1}^{\infty} c_n \zeta^n = W(\zeta)$$

on the curve $\zeta = e^{i\theta}$, therefore from the table $T'(x_j, y_j)$ versus θ_j one may write

$$x_j + iy_j = \sum_{n=1}^{\infty} c_n e^{in\theta_j}$$

Since the figure is symmetric about the x axis the c_n's are real. From symmetry about the y axis it is known that the possible nonzero c_n are c_{2m-1}, m = 1, 2, ...

$$\therefore \quad x_j + iy_j = \sum_{n=1}^{\infty} c_{2n-1} e^{i(2n-1)\theta_j} \tag{19}$$

Using Euler's formula and comparing real and imaginary parts (19) give

$$x_j = \sum_{n=1}^{N} c_{2n-1} \cos(2n-1)\theta_j$$

$$y_j = \sum_{n=1}^{N} c_{2n-1} \sin(2n-1)\theta_j$$

where N is less than the number of entries in T'.

Either of the above equations is sufficient to determine the desired c_n's by Fourier analysis using table T'. One may now write

$$W(\zeta) = \sum_{n=1}^{N} c_{2n-1} \zeta^{2n-1} \tag{20}$$

COMMENTS CONCERNING PROGRAM LISTING, TABLE OF CONFORMAL MAPPING COEFFICIENTS AND PLOTS OF CONFORMAL MAPS

Sufficient comment cards have been inserted in the conformal mapping program to make its listing self-explanatory. See pages 14 through 25. On page 25 the points and weights for 64 point Gaussian quadrature has been recorded in the format required by this program. For those who wish to use different order Gaussian quadratue see [13] pp. 916-919 or [14].

Pages 26 through 41 contain a table of the conformal mapping coefficients. By the symmetry of the figures considered the even coefficients are identically zero, hence they have not been listed. The coefficients CN(N) of the table correspond to $C_{(2n-1)}$ of (20) (page 9) where $2n-1 = N$.

The computer program generates the first 200 odd coefficients, however many of the figures were such that only a fraction of the coefficients needed to be recorded since the rest were zero. For example, for the set of figures with B/A = 1.0 the table (see pages 26,27) lists only the coefficients up to N = 159. This indicates that the coefficients N = 161 to N = 399 generated for the figures with B/A = 1.0 were all zero.

For cases where all 200 odd coefficients are recorded (i.e. figures with B/A = 3. or B/A = 1./3.) these are coefficients of a series which conformally maps the unit circle onto a figure similar to the desired figure (by virtue of the fact that the series is a truncation of the desired series). Only for the extreme cases; B/A = 3., 1./3. ; ALPHA = BETA = 1.6 (cf. note at head of listing of subroutine GETA) does the figure significantly differ from the desired figure. See discussion of conformal map plots below.

The trivial case B/A = 1.0, ALPHA = BETA = 2.0 was included in the table to give an indication of the accuracy of the program.

To give a visual check of the accuracy of the conformal maps a computer plot was made of the original figure and superimposed on it was a plot of the figure obtained by using the conformal map. Pages 44 and 45 contain several of these plots.

In general the accuracy of a conformal map for a figure with B/A = X will be comparable to the accuracy of a conformal map for a figure with B/A = 1/X.

As explained in the note at head of listing of subroutine GETA the accuracy of the generated conformal map is best for figures close to a circle, i.e. B/A close to 1.0 and ALPHA, BETA close to 2.0. The bad case B/A = 3.0, ALPHA = BETA = 1.6 seen on page 45 was included to show the limitation of the program for conformally mapping figures severely different from a circle.

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C
C
C
C     CONMAP    'CON'FORMAL 'MAP'PING COEFFICIENTS
C     A COMPUTER PROGRAM WHICH DETERMINES THE COEFFICIENTS OF THE CON-
C     FORMAL MAPPING OF THE INTERIOR OF THE UNIT CIRCLE ONTO THE INTERI-
C     OR  OF ABS(X/A)**ALPHA + ABS(Y/B)**BETA = 1 ... EQ A
C
C     THE COMMENTS IN THIS LISTING ARE NOT INTENDED TO BE AN EXHAUSTIVE
C     EXPLANATION OF ALL ASPECTS OF THE GIVEN PROBLEM.  RATHER THEY ARE
C     INCLUDED AS A GUIDE TO THOSE WHO ARE FAMILIAR WITH THE PARTICULAR
C     MATHEMATICAL METHOD USED AND THE ASSOCIATED MATHEMATICAL TOOLS
C     (SEE TEXT FOR MORE DETAILS).
C
C     NOTE....    FOR LIMITATIONS ON ALPHA, BETA, AND B/A SEE NOTE AT
C     THE HEAD OF THE LISTING OF SUBROUTINE GETA
C
C
C     MAIN PROGRAM
C
      DIMENSION GP(64), GW(64), GX(64), GY(64),CN( 500)
      COMMON ALPHA, BETA, P, GP, GW, GX, GY
      READ (5,1000) (GP(I),I=1,32),(GW(I),I=1,32)
C
C     GP = GAUSSIAN POINTS
C     64 POINTS ARE USED IN THIS PROGRAM.  I.E. GP(I), -GP(I)   I=1,32
C     GP(1) = .999305 ... ETC.
C     GW = GAUSSIAN WEIGHTS
C
 1000 FORMAT (4F20.0)
    5 READ (5,2000, END = 5000) CS , ALPHA , BETA
      WRITE (6,6000) CS,ALPHA,BETA
C
C     CS = B/A   'C'OEFFICIENT OF 'S'LENDERNESS
C     AS IS APPARENT FROM EQ A THE SHAPE OF THE CURVE (EQ A) IS NOT
C     EFFECTED BY THE PARAMETERS A AND B SEPARATELY BUT BY THEIR RATIO.
C     CONSEQUENTLY THE CONFORMAL MAPPING COEFFICIENTS ARE DETERMINED TO
C     WITHIN A MULTIPLICATIVE CONSTANT WHEN ONLY B/A IS SPECIFIED.  TO
C     OBTAIN THE CONFORMAL MAPPING COEFFICIENTS FOR A PARTICULAR SIZE
C     CURVE MERELY MULTIPLY THE GENERATED COEFFICIENTS BY A.
C
 6000 FORMAT (1H1,86H THE FIRST  50 COEFFICIENTS OF THE CONFORMAL MAPPIN
     1G OF THE INTERIOR OF THE CURVE WITH,/14H A = 1.00, B =,F5.2,8H ALP
     2HA =,F5.2,7H BETA =,F5.2,42H ONTO THE INTERIOR OF THE UNIT CIRCLE
     3ARE ///)
 2000 FORMAT (3F10.4)
C
C     GARL    'G'AUSSIAN 'AR'C'L'ENGTH
C     AS IS READILY SEEN FROM EQ A THE CURVE IS SYMMETRIC ABOUT BOTH
C     THE X AND Y AXES.  FOR THIS REASON WE NEED ONLY INFORMATION ABOUT
C     THE CURVE IN ONE QUADRANT( THE FIRST QUADRANT IS USED ).
C     THE PARTICULAR MATHEMATICAL METHOD USED TO DETERMINE THE CONFORMAL
C     MAPPING ULTIMATELY REDUCES TO THE DETERMINATION OF THE SOLUTION TO
C     AN INTEGRAL EQUATION (SEE TEXT).  IN THIS PROGRAM GAUSSIAN QUAD-
C     RATURE IS EMPLOYED TO REDUCE THE INTEGRAL EQUATION TO A SYSTEM OF
C     LINEAR ALGEBRAIC EQUATIONS.
C     THE SUBROUTINE GARL DETERMINES THE ARCLENGTH,'ARCL', OF THE
C     GIVEN CURVE IN A QUADRANT. THE SUBROUTINE  THEN DETERMINES THE
C     POINTS,(GX(I),GY(I)), ON THE CURVE IN THE FIRST QUADRANT WHICH ARE
C     SPACED ALONG THE ARCLENGTH OF THE CURVE PROPORTIONALLY TO THE
C     GAUSSIAN POINTS GP(I).    I.E. IF AL(I) ARE THE ARCLENGTHS MEASURED
C     FROM AN INTERCEPT TO POINT(GX(I),GY(I)) WE HAVE THE RELATION
C     AL(I) = (ARCLENGTH/2)(GP'(I) + 1) WHERE GP'(I) IS THE FULL ARRAY
C     OF GAUSSIAN POINTS FROM 1 TO -1,GP'(I=1,32) = GP(I=1,32), GP'(I=
C     33,64) = -GP(J=1,32  J=65 - I)
C
      CALL GARL ( CS,ARCL)
      DO 10 I=33,64
```

```
      M  = 65 - I
   10 GW(I) = GW(M)
C
C     GETA    'GE'NERATES A 'TA'BLE OF(GX(I),GY(I)) VERSUS E(I) WHERE
C     E(I) ARE THE ANGLES THE RADII TO THE IMAGE POINTS OF (GX(I),GY(I))
C     ON THE UNIT CIRCLE MAKE WITH THE ABSCISSA IN THE IMAGE PLANE.
C     THIS TABLE IS THEN MADE SUITABLE FOR FOURIER ANALYSIS WHICH IN
C     TURN YIELDS THE DESIRED COEFFICIENTS, CN(I), FOR THE CONFORMAL
C     MAPPING.
C
      CALL GETA (CN,ARCL)
      WRITE (6,3000) (CN(I),I=1,399,2)
 3000 FORMAT (T50,F9.4)
C
C     FORMAT F9.4 IS ACHIEVED BY ROUNDING-OFF NOT BY TRUNCATION
C
      WRITE (7,4000) (CN(I),I=1,399,2)
 4000 FORMAT(8F9.4)
      GO TO 5
 5000 CALL SYSERR
      END
C
C
C     GARL    'G'AUSSIAN 'AR'C'L'ENGTH
C     AS IS READILY SEEN FROM EQ A THE CURVE IS SYMMETRIC ABOUT BOTH
C     THE X AND Y AXES.  FOR THIS REASON WE NEED ONLY INFORMATION ABOUT
C     THE CURVE IN ONE QUADRANT( THE FIRST QUADRANT IS USED ).
C     THE PARTICULAR MATHEMATICAL METHOD USED TO DETERMINE THE CONFORMAL
C     MAPPING ULTIMATELY REDUCES TO THE DETERMINATION OF THE SOLUTION TO
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C     SPACED ALONG THE ARCLENGTH OF THE CURVE PROPORTIONALLY TO THE
C     GAUSSIAN POINTS GP(I).    I.E. IF AL(I) ARE THE ARCLENGTHS MEASURED
C     FROM AN INTERCEPT TO POINT(GX(I),GY(I)) WE HAVE THE RELATION
C     AL(I) = (ARCLENGTH/2)(GP'(I) + 1) WHERE GP'(I) IS THE FULL ARRAY
C     OF GAUSSIAN POINTS FROM 1 TO -1,GP'(I=1,32) = GP(I=1,32), GP'(I=
C     33,64) = -GP(J=1,32  J=65 - I)
C
      SUBROUTINE GARL ( CS,ARCL)
      COMMON ALPHA, BETA, P, GP, GW, GX, GY
      DIMENSION AL(64), GP(64), GW(64), GX(64), GY(64)
      EXTERNAL FUNX , FUNY
      XISF(Y) = (ABS( 1. - P*Y**BETA))**(1./ALPHA)
C
C     XISF(Y) IS EQ A SOLVED FOR X IN TERMS OF Y
C
      YISF(X) = (ABS( 1. - X**ALPHA)/P)**(1./BETA)
C
C     YISF(X) IS EQ A SOLVED FOR Y IN TERMS OF X
C
      P  =  (1./CS)**BETA
      N  = 32
C
C     THE FOLLOWING IF STATEMENTS DIVIDE THE POSSIBLE CURVES INTO VAR-
C     IOUS CLASSES
C     CLASS 1   ALPHA = BETA = 1.
C     CLASS 2   ALPHA.LE.1. , BETA.LT.1.
C     CLASS 3   ALPHA.LT.1. , BETA.EQ.1.
C     CLASS 4   ALPHA.GE.1. , BETA.GT.1.
C     CLASS 5   ALPHA.GT.1. , BETA.EQ.1.
C     CLASS 6   ALPHA.LT.1. , BETA.GT.1.
C     CLASS 7   ALPHA.GT.1. , BETA.LT.1.
      IF (ALPHA - 1.)5,10,15
    5 IF (BETA - 1.)35,45,65
   10 IF (BETA - 1.)35,20,50
   15 IF (BETA - 1.)70,60,50
```

```
C
C     ARCLENGTH IS DETERMINED FOR CLASS 1 CURVES
C
   20 ARCL = SQRT(1. + (CS)**2)
C
C     THE ARCLENGTHS NEEDED FOR DETERMINING THE POINTS FOR GAUSSIAN
C     QUADRATURE ARE DETERMINED FOR CLASS 1 CURVES
C
      HA = ARCL/2.
      DO 25 J=1,32
   25 AL(J) = HA*(-GP(J)) + HA
      DO 30 J=1,32
      M  = 65 - J
   30 AL(M) = HA*GP(J) + HA
      GO TO 75
C
C     ARCLENGTH IS DETERMINED FOR CLASS 2 CURVES
C
   35 F1 = CS
   40 X  = 1.
      DELTA = .01
C
C     IN INTEGRATING FOR THE ARCLENGTH IT IS AT MOST TIMES MORE ADVAN-
C     TAGEOUS TO USE X OR Y AS THE INDEPENDENT VARIABLE DEPENDING ON
C     WHERE ONE IS ON THE CURVE.   CRITP ('CRIT'ICAL 'P'OINT) DETERMINES
C     THE POINTS ON THE CURVE AT WHICH THE SLOPE IS -B/A.    ON ONE SIDE
C     OF THESE POINTS WE WILL USE X AS THE INDEPENDENT VARIABLE TO
C     DETERMINE ARCLENGTH AND ON THE OTHER SIDE WE WILL USE Y AS THE
C     INDEPENDENT VARIABLE.
C
      CALL CRITP (X,F1,CS,DELTA)
      R  = X
      S  = 1.
      X1 = X
      Y1 = YISF(X)
      IC = 0
      X2 = 0.0
      Y2 = CS
C
C     SINTG   'S'UBROUTINE FOR 'INT'EGRATION BY 'G'AUSSIAN QUADRATURE
C     THIS SUBROUTINE USES GAUSSIAN QUADRATURE TO INTEGRATE FOR THE
C     ARCLENGTH
C     FUNX IS THE INTEGRAND OF THE ARCLENGTH INTEGRAL IN TERMS OF X
C
      CALL SINTG (FUNX,R,S,N,ARCL1)
      R  = YISF(X)
      S  = CS
C
C     FUNY IS THE INTEGRAND OF THE ARCLENGTH INTEGRAL IN TERMS OF Y
C
      CALL SINTG (FUNY,R,S,N,ARCL2)
      ARCL = ARCL1 + ARCL2
      GO TO 85
C
C     ARCLENGTH IS DETERMINED FOR CLASS 3 CURVES
C
   45 F1 = (-ALPHA/BETA + 1.)*CS
      GO TO 40
C
C     ARCLENGTH IS DETERMINED FOR CLASS 4 CURVES
C
   50 F1 = -10.**20
   55 X  = 1.
      DELTA = .01
      CALL CRITP (X,F1,CS,DELTA)
      R  = 0.0
      S  = YISF(X)
      X1 = X
      Y1 = S
```

```
      IC = 1
      X2 = 0.0
      Y2 = CS
      CALL SINTG (FUNY,R,S,N,ARCL1)
      R  = 0.0
      S  = X
      CALL SINTG (FUNX,R,S,N,ARCL2)
      ARCL = ARCL1 + ARCL2
      GO TO 85
C
C     ARCLENGTH IS DETERMINED FOR CLASS 5 CURVES
C
   60 F1 = (-ALPHA/BETA + 1.)*CS
      GO TO 55
C
C     ARCLENGTH IS DETERMINED FOR CLASS 6 CURVES
C
   65 F1 = -10.**20
      X  = 1.0
      DELTA = .01
      CALL CRITP (X,F1,CS,DELTA)
      R  = 0.0
      S  = YISF(X)
      Y1 = S
      CALL SINTG (FUNY,R,S,N,ARCL1)
      X1 = X
      X  = 0.0
      DELTA = -.01
      CALL CRITP (X,F1,CS,DELTA)
      R  = YISF(X)
      Y2 = R
      X2 = X
      IC = 1
      S  = CS
      CALL SINTG (FUNY,R,S,N,ARCL3)
      R  = X
      S  = X1
      CALL SINTG (FUNX,R,S,N,ARCL2)
      ARCL = ARCL1 + ARCL2 + ARCL3
      GO TO 85
C
C     ARCLENGTH IS DETERMINED FOR CLASS 7 CURVES
C
   70 F1 = CS
      X  = 1.0
      DELTA = .01
      CALL CRITP (X,F1,CS,DELTA)
      R  = X
      S  = 1.0
      CALL SINTG (FUNX,R,S,N,ARCL1)
      Y1 = YISF(X)
      X1 = X
      X  = 0.0
      DELTA = -.01
      CALL CRITP (X,F1,CS,DELTA)
      R  = 0.0
      S  = X
      CALL SINTG (FUNX,R,S,N,ARCL3)
      S  = YISF(X)
      R  = Y1
      X2 = X
      Y2 = S
      IC = 0
      CALL SINTG (FUNY,R,S,N,ARCL2)
      ARCL = ARCL1 + ARCL2 + ARCL3
      GO TO 85
C
C     THE POINTS NEEDED FOR GAUSSIAN QUADRATURE ARE DETERMINED FOR CLASS
```

```
C     1 CURVES
C
   75 DO 80 I=1,64
      GX(I) = 1. - AL(I)/ARCL
   80 GY(I) = AL(I)*CS/ARCL
      GO TO 180
C
C     THE ARCLENGTHS NEEDED FOR DETERMINING THE POINTS FOR GAUSSIAN
C     QUADRATURE ARE DETERMINED FOR CURVE CLASSES 2 THROUGH 7
C
   85 HA = ARCL/2.
      DO 90 J=1,32
   90 AL(J) = HA*(-GP(J)) + HA
      DO 95 J=1,32
      M  = 65 - J
   95 AL(M) = HA*GP(J) + HA
C
C     THE REST OF THIS SUBROUTINE IS DEVOTED TO DETERMINING THE POINTS
C     NEEDED FOR GAUSSIAN QUADRATURE FOR CURVE CLASSES 2 THROUGH 7.
C     THE PROGRAM FLOW IS DESIGNED EXCLUSIVELY FOR THE PURPOSE OF US-
C     ING FUNX OR FUNY DEPENDING ON WHAT PART OF THE CURVE ONE IS SITUATED.
C
      I  = 1
      Y  = 0.0
      X  = 1.0
      SI = AL(1)
      GO TO 102
  100 SI = AL(I) - AL(I-1)
  102 IF (AL(I) - ARCL1)105,120,125
  105 IF (IC)180,110,115
  110 S  = X
      R  = X
C
C     CALCX 'CALC'ULATES THE'X' COORDINATE OF THE POINT ON THE CURVE
C     THAT LIES AT A GIVEN ARCLENGTH FROM THE X INTERCEPT
C
      CALL CALCX (FUNX,R,S,N,X,SI)
C
C     YISF(X) IS EQ A SOLVED FOR Y IN TERMS OF X
C
      Y  = YISF(X)
      GO TO 175
  115 R  = Y
      S  = Y
C
C     CALCY 'CALC'ULATES THE 'Y' COORDINATE OF THE POINT ON THE CURVE
C     THAT LIES AT A GIVEN ARCLENGTH FROM THE X INTERCEPT.
C
      CALL CALCY (FUNY,R,S,N,Y,SI,CS)
C
C     XISF IS EQ A SOLVED FOR X IN TERMS OF Y
C
      X  = XISF(Y)
      GO TO 175
  120 X  = X1
      Y  = Y1
      GO TO 175
  125 IF (AL(I) - ARCL1 - ARCL2)130,145,150
  130 IF (IC)180,135,140
  135 S  = Y
      R  = Y
      CALL CALCY (FUNY,R,S,N,Y,SI,CS)
      X  = XISF(Y)
      GO TO 175
  140 R  = X
      S  = X
      CALL CALCX (FUNX,R,S,N,X,SI)
      Y  = YISF(X)
```

```
      GO TO 175
  145 X   = X2
      Y   = Y2
      GO TO 175
  150 IF (AL(I) - ARCL1 - ARCL2 - ARCL3)155,170,180
  155 IF (IC)180,160,165
  160 S   = X
      R   = X
      CALL CALCX (FUNX,R,S,N,X,SI)
      Y   = YISF(X)
      GO TO 175
  165 S   = Y
      R   = Y
      CALL CALCY (FUNY,R,S,N,Y,SI,CS)
      X   = XISF(Y)
      GO TO 175
  170 X   = 0.0
      Y   = CS
  175 GX(I) = X
      GY(I) = Y
      I   = I + 1
      IF (I - 64)100,100,180
  180 CONTINUE
      RETURN
      END
C
C
C     IN INTEGRATING FOR THE ARCLENGTH IT IS AT MOST TIMES MORE ADVAN-
C     TAGEOUS TO USE X OR Y AS THE INDEPENDENT VARIABLE DEPENDING ON
C     WHERE ONE IS ON THE CURVE.   CRITP ('CRIT'ICAL 'P'OINT) DETERMINES
C     THE POINTS ON THE CURVE AT WHICH THE SLOPE IS -B/A.    ON ONE SIDE
C     OF THESE POINTS WE WILL USE X AS THE INDEPENDENT VARIABLE TO
C     DETERMINE ARCLENGTH AND ON THE OTHER SIDE WE WILL USE Y AS THE
C     INDEPENDENT VARIABLE.
C
      SUBROUTINE CRITP (X,F1,CS,DELTA)
      COMMON ALPHA, BETA
      F2 = F1
    5 X   = X - DELTA
      F   = -ALPHA*CS/BETA*X**(ALPHA - 1.)*(1. - X**ALPHA)**(1./BETA - 1.
     1) + CS
      IF (F/F2)10,35,35
   10 IF (ABS( F) -.00001)40,40,15
   15 X   = X + DELTA
   20 DELTA = DELTA/2.
      X   = X - DELTA
      F   = -ALPHA*CS/BETA*X**(ALPHA - 1.)*(1. - X**ALPHA)**(1./BETA - 1.
     1) + CS
      IF (F/F2)25,30,30
   25 IF (ABS( F) -.00001)40,40,15
   30 F2 = F
      GO TO 20
   35 F2 = F
      GO TO 5
   40 RETURN
      END
C
C
C     SINTG   'S'UBROUTINE FOR 'INT'EGRATION BY 'G'AUSSIAN QUADRATURE
C     THIS SUBROUTINE USES GAUSSIAN QUADRATURE TO INTEGRATE FOR THE
C     ARCLENGTH
C
      SUBROUTINE SINTG (FX,R,S,N,ARCL)
      DIMENSION GP(64), GW(64)
      COMMON ALPHA, BETA, P, GP, GW
      SPR2 = (S + R)/2.
      SMR2 = (S - R)/2.
      ARCL = 0.0
      DO 5 I=1,N
```

```
      U   = SMR2*GP(I) + SPR2
      V   = -SMR2*GP(I) + SPR2
    5 ARCL = ARCL + GW(I)*(FX(U) + FX(V))
      ARCL = ARCL*SMR2
      RETURN
      END
C
C
C     FUNX IS THE INTEGRAND OF THE ARCLENGTH INTEGRAL IN TERMS OF X
C
      FUNCTION FUNX(X)
      COMMON ALPHA, BETA, P
      FUNX   = SQRT(1. + ((ALPHA/(BETA*P))**2)*(X**(2.*(ALPHA - 1.)))*(AB
     1S( (1. - X**ALPHA)/P))**(2.*(1./BETA - 1.)))
      RETURN
      END
C
C
C     FUNY IS THE INTEGRAND OF THE ARCLENGTH INTEGRAL IN TERMS OF Y
C
      FUNCTION FUNY(Y)
      COMMON ALPHA, BETA, P
      FUNY   = SQRT(1. + ((BETA*P/ALPHA)**2)*(Y**(2.*(BETA - 1.)))*((ABS(
     1 1. - P*Y**BETA))**(2.*(1./ALPHA - 1.))))
      RETURN
      END
C
C
C     CALCX 'CALC'ULATES THE'X' COORDINATE OF THE POINT ON THE CURVE
C     THAT LIES AT A GIVEN ARCLENGTH FROM THE X INTERCEPT
C
      SUBROUTINE CALCX (FX,R,S,N,X,SI)
      DELT = -.1
      ARP = 0.0
      RP = R
      RA = 0.0
      ARAH = SI
    5 R   = R + DELT
      IF (R)10,10,15
   10 R   = 0.0
   15 CALL SINTG (FX,R,S,N,ARCLO)
      IF (SI - ARCLO)30,40,20
   20 IF (ABS( SI - ARCLO) -.00001)40,40,25
   25 ARP = ARCLO
      RP = R
      R   = RP + (RA - RP)*(SI - ARP)/(ARAH - ARP)
      GO TO 15
   30 IF (ABS( SI - ARCLO) -.00001)40,40,35
   35 ARAH = ARCLO
      RA = R
      R   = RP + (RA - RP)*(SI - ARP)/(ARAH - ARP)
      GO TO 15
   40 X   = R
      RETURN
      END
C
C
C     CALCY 'CALC'ULATES THE 'Y' COORDINATE OF THE POINT ON THE CURVE
C     THAT LIES AT A GIVEN ARCLENGTH FROM THE X INTERCEPT.
C
      SUBROUTINE CALCY (FY,R,S,N,Y,SI,CS)
      DELT = .1
      ARP = 0.0
      SP = S
      SA = CS
      ARAH = SI
    5 S   = S + DELT
      IF (CS - S)10,10,15
   10 S   = CS
```

```
   15 CALL SINTG (FY,R,S,N,ARCLO)
      IF (SI - ARCLO)30,40,20
   20 IF (ABS( SI - ARCLO) -.00001)40,40,25
   25 ARP = ARCLO
      SP = S
      S  = SP + (SA - SP)*(SI - ARP)/(ARAH - ARP)
      GO TO 15
   30 IF (ABS( SI - ARCLO) -.00001)40,40,35
   35 ARAH = ARCLO
      SA = S
      S  = SP + (SA - SP)*(SI - ARP)/(ARAH - ARP)
      GO TO 15
   40 Y  = S
      RETURN
      END
C
C
C     GETA    'GE'NERATES A 'TA'BLE OF(GX(I),GY(I)) VERSUS E(I) WHERE
C     E(I) ARE THE ANGLES THE RADII TO THE IMAGE POINTS OF (GX(I),GY(I))
C     ON THE UNIT CIRCLE MAKE WITH THE ABSCISSA IN THE IMAGE PLANE.
C     THIS TABLE IS THEN MADE SUITABLE FOR FOURIER ANALYSIS WHICH IN
C     TURN YIELDS THE DESIRED COEFFICIENTS, CN(I), FOR THE CONFORMAL
C     MAPPING.
C
C
C     NOTE - THIS SUBROUTINE WILL NOT IN ITS PRESENT FORM HANDLE CURVES,
C     EQ A,WITH ALPHA AND/OR BETA SUCH THAT THE CURVE HAS ANGULAR POINTS
C     I.E. ALPHA AND/OR BETA .LE. 1.
C
C     THIS SUBROUTINE YIELDS ACCURATE RESULTS FOR ALPHA AND/OR BETA
C     DOWN TO 1.6 .   THE TOLERANCES IN THE SUBROUTINE ARE SET SO THAT
C     THIS SUBROUTINE FUNCTIONS ACCURATELY FOR ALPHA AND/OR BETA UP TO
C     10.0.   (ALPHA = BETA = 10.0 IS ESSENTIALLY A RECTANGLE.)  FOR
C     ALPHA AND/OR BETA.GT.10.0 THE TOLERANCES MAY HAVE TO BE ADJUSTED.
C     THE TABLE (GX(I),GY(I)) VERSUS E(I) MUST BE SUCH THAT AS GX(I)
C     STRICTLY MONOTONICALLY DECREASES FROM 1. TO 0.0 E(I) MUST STRICTLY
C     MONOTONICALLY INCREASE FROM 0.0 TO PIE/2.0 .   AS ALPHA AND/OR
C     BETA GOES FROM 1.6 TO 1.0 THE TABLE GENERATED BECOMES INCREASINGLY
C     LESS ACCURATE BY REASON THAT THE ABOVE MENTIONED MONOTONICITY IS
C     NOT OBTAINED IN THE BEGINNING AND END OF THE TABLE.   THIS LACK
C     OF MONOTONICITY AT THE ENDS OF THE TABLE HOWEVER DOES NOT EXTEND
C     FAR INTO THE TABLE.  FOR EXAMPLE, FOR THE EXTREME CASE OF ALPHA =
C     BETA = 1. ( B/A=1.) ONLY THE FIRST FOUR ENTRIES AND THE LAST SIX
C     ENTRIES OF THE 64 ENTRY TABLE ARE DEFECTIVE BY REASON OF NOT
C     SATISFYING  THE MONOTONICITY REQUIREMENT.  IT IS SUGGESTED THAT
C     THOSE WHO ARE INTERESTED IN THE RANGE 1.LE. ALPHA AND/OR BETA .LT.
C     1.6 THAT IF ALPHA AND/OR BETA LIES IN THIS RANGE THE TABLE
C     (GX(I),GY(I)) VERSUS E(I) BE TRANSFERRED TO A SUBROUTINE THAT WILL
C     DETERMINE THE DEGREE OF LACK OF MONOTONICITY AND SUPPLY NEW
C     ENTRIES FOR THE ENDS OF THE TABLE EITHER BY EXTRAPOLATION FROM THE
C     INTERIOR OF THE TABLE OR BY SOME OTHER METHOD.
C
C     THE ORIGINAL TABLE CAN BE IMPROVED SLIGHTLY,AT THE EXPENSE OF
C     INCREASED EXECUTION TIME, BY USING MORE GAUSSIAN POINTS.  FOR EX-
C     AMPLE FOR THE ABOVE MENTIONED CASE ( ALPHA=BETA=B/A= 1.0) USING
C     96 POINT GAUSSIAN QUADRATURE WE GENERATE A 96 ENTRY TABLE OF WHICH
C     THE FIRST FIVE ENTRIES AND LAST THREE ENTRIES WERE FAULTY BY REA-
C     SON OF NOT SATISFYING THE MONOTONICITY REQUIREMENT.
C
C     USE OF DOUBLE PRECISION YIELDED NO SIGNIFICANT IMPROVEMENT.
C
C     AS THIS SUBROUTINE STANDS IT GIVES ACCURATE RESULTS FOR 1/3.LE.B/A
C     .LE. 3.  SMALL ENLARGEMENTS OF THIS RANGE OF B/A ARE GAINED IF
C     MORE GAUSSIAN POINTS ARE USED.  FOR OUR PURPOSES 64 POINT GAUSSIAN
C     QUADRATURE WAS OPTIMAL CONSIDERING PROGRAM EXECUTION TIME, RANGE
C     OF B/A IN WHICH SUFFICIENT ACCURACY IS ACHIEVED AND THE NUMBER OF
C     SIGNIFICANT COEFFICIENTS OBTAINED FOR THE CONFORMAL MAPPING.  FOR
C     THE RANGE .5.LE. B/A.LE. 2. OFTEN 50 OR LESS NON-ZERO ( TO FOUR
C     -PLACE ACCURACY) COEFFICIENTS ARE OBTAINED (I.E. THE POLYNOMIAL
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C     REPRESENTING THE CONFORMAL MAPPING IS OF DEGREE 99 OR LESS).  (BY
C     SYMMETRY THE COEFFICIENTS OF EVEN ORDER TERMS ARE IDENTICALLY ZERO
C     THEREFORE THE COMPUTER PROGRAM GENERATES ONLY COEFFICIENTS FOR ODD
C     ORDER TERMS).
C     FOR THE RANGE 1./3..LE.B/A.LE.3.USUALLY 200 OR LESS NON-ZERO
C     COEFFICIENTS ARE OBTAINED (CONFORMAL MAPPING POLYNOMIAL IS OF
C     DEGREE 399 OR LESS).  SINCE MANY METHODS INVOLVING CONFORMAL
C     MAPPING BECOME IMPRACTICAL FOR SHAPES WHOSE CONFORMAL MAPPING
C     POLYNOMIAL IS OF HIGH DEGREE WE CHOSE TO STOP DEVELOPING THE
C     PROGRAM WHEN THE RANGE 1./3.LE. B/A.LE.3. WAS OBTAINED SINCE THIS
C     RANGE IS ALREADY INVOLVED WITH CONFORMAL MAPPING POLYNOMIALS OF
C     DEGREE 399.
C     AS A RULE OF THUMB THE NUMBER OF NON-ZERO COEFFICIENTS INCREASE
C     AS B/A MOVES FROM 1. AND/OR AS ALPHA AND/OR BETA MOVES FROM 2.0.
C
C     AS B/A PROGRESSES FROM THE RANGE .25.LE.B/A.LE.4. THE TABLE
C     (GX(I),GY(I)) VERSUS E(I) BECOMES INCREASINGLY FAULTY BY REASON OF
C     NOT SATISFYING THE MONOTONICITY REQUIREMENT.
C     ALL COMMENTS MADE CONCERNING ACCURACY AND IMPROVEMENT OF TABLES
C     FOR 1.0.LE. ALPHA AND/OR BETA.LT.1.6 APPLY EQUALLY WELL FOR TABLES
C     WITH B/A OUTSIDE THE RANGE .25.LE.B/A.LE.4.0.
C
      SUBROUTINE GETA (CN,ARCL)
      DIMENSION PHI(64), CURVE(64),THETA(64,65),  E(64),CN( 500),
     IGP(64), GW(64), GX(64), GY(64), XI(500), AN(500)
      COMMON ALPHA, BETA, P, GP, GW, GX, GY
C
C     UNDERZ('OFF') CAUSES UNDERFLOW ERROR MESSAGES NOT TO BE WRITTEN.
C     FOR SOME CURVES, SAY B/A = 2. , ALPHA = BETA = 10. , THE COMPUTER
C     PROGRAM UNDERFLOWS IN THE PROCESS OF DETERMINING THE CURVATURE
C     ( SEE CURVE(I) BELOW ) AT VARIOUS POINTS ON THE CURVE.   SINCE THE
C     UNDERFLOW DOES NOT INVALIDATE THE COMPUTATIONS MADE IN DETERMINING
C     THE CURVATURE THE FACT THAT A UNDERFLOW MAY OCCUR IS OF NO CONSE-
C     QUENCE.   UNDERZ('OFF') , OF COURSE, SHOULD BE DELETED IN THE
C     DEVELOPMENTAL STAGE OF ANY MODIFICATION OF THIS SUBROUTINE.
C
      CALL UNDERZ('OFF')
C
C     IN THE COURSE OF SETTING UP THE SYSTEM OF LINEAR EQUATIONS WHICH
C     REPLACE THE INTEGRAL EQUATION ( SEE TEXT ) WE NEED TO DETERMINE AT
C     EACH POINT ON THE CURVE USED IN THE GAUSSIAN QUADRATURE,
C     (GX(I),GY(I)), THE ANGLE THE OUTWARD NORMAL MAKES WITH THE POSI-
C     TIVE X-AXIS,PHE(I), AND THE CURVATURE, CURVE(I).   THE FOLLOWING
C     SECTION OF THE SUBROUTINE COMPUTES THESE VALUES.
C
      DO 20 I=1,64
      X  = GX(I)
      IF (1./BETA - 1.)5,15,15
    5 IF (ABS( 1. - X**ALPHA) -.0000001)10,15,15
   10 PHI(I) = 0.0
      CURVE(I) = 0.0
      GO TO 20
   15 DYX = -ALPHA*X**(ALPHA - 1.)*(ABS( 1. - X**ALPHA)/P)**(1./BETA - 1
     1.)/(BETA*P)
      PHI(I) = 1.57079632 + ATAN(DYX)
      DDYX = -ALPHA*(ALPHA - 1.)*X**(ALPHA - 2.)*(ABS( 1. - X**ALPHA)/P)
     1**(1./BETA - 1.)/(BETA*P) + ALPHA**2*(1./BETA - 1.)*X**(2.*ALPHA -
     2 2.)*(ABS( 1. - X**ALPHA)/P)**(1./BETA - 2.)/(BETA*P**2)
      IF(ABS( DYX)-0.10E+14) 18,18,16
   16 CURVE(I) = 0.0
      GO TO 20
   18 CURVE(I) = ABS( DDYX)/((1. + DYX**2)**1.5)
   20 CONTINUE
C
C     THE FOLLOWING SECTION GENERATES THE COEFFICIENTS, THETA(I,J), OF
C     THE LINEAR SYSTEM OF EQUATIONS.
C
      CONST = 0.0
      DO 22 J=1,65
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```
      DO 22 I = 1,64
   22 THETA(I,J) = 0.0
      DO 55 N=1,64
      DO 50 I=1,64
      PHE = PHI(I)
      IF (I - N)35,30,35
C
C     G1,G2,G3AND G4 ARE THE KERNELS OF THE INTEGRAL EQUATION IN THE
C     FIRST, SECOND, THIRD, AND FOURTH QUADRANTS RESPECTIVELY.
C
   30 G1 =+.5*CURVE(N)
      GO TO 40
   35 G1 = ((GX(I) -GX(N))*COS(PHE) + (GY(I) - GY(N))*SIN(PHE))/((GX(I)
     1- GX(N))**2 + (GY(I) - GY(N))**2)
   40 G2 = ((GX(I) - GX(N))*COS(PHE) + (GY(I) + GY(N))*SIN(PHE))/((GX(I)
     1- GX(N))**2 + (GY(I) + GY(N))**2)
      G3 = ((GX(I) + GX(N))*COS(PHE) + (GY(I) + GY(N))*SIN(PHE))/((GX(I)
     1 + GX(N))**2 + (GY(I) + GY(N))**2)
      G4 = ((GX(I) + GX(N))*COS(PHE) + (GY(I) - GY(N))*SIN(PHE))/((GX(I)
     1+ GX(N))**2 + (GY(I) - GY(N))**2)
      THETA(N,I) = -ARCL*GW(I)*(G1 - G2 + G3 - G4)/(2.*3.14159265)
      CON = ARCL*(G3 + G4 + 2.*G2)*GW(I)/2.
      CONST = CONST + CON
      IF (I - N)50,45,50
   45 THETA(N,I) = THETA(N,I) + 1.
   50 CONTINUE
      THETA(N,65) = CONST
   55 CONST = 0.0
      DO 65 N= 1,64
C
C     ATA1 IS THE ANGLE THE POSITION VECTOR OF (GX(I),GY(I)) MAKES WITH
C     THE POSITIVE X-AXIS.
C
      ATA1 = ATAN(GY(N)/GX(N))
      IF (ABS( 1. - GX(N)) -.0000001)57,58,58
   57 ATA2 = 3.14159265/2.
      GO TO 60
C
C     ATA2 IS THE ANGLE THE VECTOR FROM POINT (1,0) TO POINT
C     (GX(I),GY(I)) MAKES WITH THE NEGATIVE X-AXIS.
C
   58 ATA2 = ATAN(GY(N)/(1. - GX(N)))
C
C     LET A TRIANGLE HAVE VERTICES (0,0),(GX(I),GY(I)), AND (1,0).
C     ATA1 AND ATA2 ARE INTERIOR ANGLES.   BN IS THE THIRD INTERIOR
C     ANGLE.  ( SEE TEXT.)
C
   60 BN = 3.14159265 - ATA1 - ATA2
 2000 FORMAT (E20.8)
   65 THETA(N,65) = THETA(N,65) - 2.*BN
      NO = 64
C
C     SIMEQ IS A STANDARD SIMULTANEOUS EQUATION SOLVER.   E(I) IS THE
C     SOLUTION.
C
      CALL SIMEQ (THETA,E,NO,MISCUE)
      IF (MISCUE - 1)75,78,70
   70 WRITE(6,4000 )
 4000 FORMAT (16H SINGULAR ANSWER)
   75 WRITE(6,5000)
 5000 FORMAT (5H GOOF)
   78 CONTINUE
C
C     THE TABLE (GX(I),GY(I)) VERSUS E(I) GENERATED THUS FAR ALTHOUGH
C     HIGHLY ACCURATE IS UNEVENLY SPACED AND COURSE, THUS IN ITS PRESENT
C     FORM THE TABLE IS UNSUITED FOR FOURIER ANALYSIS.   THIS NEXT
C     SECTION GENERATES A FINER, EVENLY SPACED TABLE XI(I) VERSUS AN(I),
C     XI AND AN CORRESPONDING TO GX AND E RESPECTIVELY.
```

```
C
      DT  =  .00314159265
      XI(1) = 1.00
      ANG = 0.0
      AN(1) = 0.0
   79 M  =  1
      DO  80 I = 2, 500
      ANG = ANG + DT
      AN(I) = ANG
  795 IF(ANG .LT. E(M +1)) GO TO 796
      M  =  M + 1
      GO TO 795
C
C     INTERP PERFORMS A SIXTH DEGREE LAGRANGE INTERPOLATION.
C
  796 CALL INTERP(ANG,E,GX,XID,M)
   80 XI(I)  =  XID
C
C     THIS LAST SECTION PERFORMS A FOURIER ANALYSIS ON THE TABLE XI(I)
C     VERSUS AN(I) TO GENERATE THE DESIRED CONFORMAL MAPPING
C     COEFFICIENTS, CN(I).
C
      DO  90 I = 1, 399,2
      A  = I
      CNI =  0.5
      DO  85 K = 2, 500
   85 CNI = CNI + XI(K)*COS(A*AN(K))
   90 CN(I) = CNI/250.
      RETURN
      END
C
C
C     A STANDARD SIMULTANEOUS EQUATIONS SOLVER
C
      SUBROUTINE SIMEQ (D,A,NO,MISCUE)
      DIMENSION D(64,65), A(64)
    1 I = NO
    2 J = NO + 1
    3 IA = I - 1
    4 DO 23 N = 1, IA
    5 DMAX = 0.0
    6 DO 10 K = N, I
    7 IF (ABS( DMAX) - ABS( D(K,N))) 8, 8, 10
    8 DMAX = D(K,N)
    9 M = K
   10 CONTINUE
   11 IF (DMAX) 14, 12, 14
   12 MISCUE = 2
   13 RETURN
   14 MISCUE = 1
   15 DO 18 L = N, J
   16 DT = D(M,L)
   17 D(M,L) = D(N,L)
   18 D(N,L) = DT
   19 IB = N + 1
   20 DO 23 L = IB, I
   21 CONST = -D(L,N)/D(N,N)
   22 DO 23 K = N, J
   23 D(L,K) = CONST*D(N,K) + D(L,K)
   24 A(I) = D(I,J)/D(I,I)
   25 IC = I - 1
   26 DO 32 N = 1, IC
   27 L = I - N
```

```
   28 SUM = 0.0
   29 ID = L + 1
   30 DO 31 K = ID, I
   31 SUM = SUM + A(K)*D(L,K)
   32 A(L) = (D(L,J) - SUM)/D(L,L)
   33 RETURN
      END
C
C
C     INTERG PERFORMS A SIXTH DEGREE LAGRANGE INTERPOLATION.
C
      SUBROUTINE INTERP(ANG,E,GX,XID,M)
      DIMENSION E(64),GX(64),DDEN(64),DNUM(64)
      IF( M.GE. 3) GO TO 5
      M  =  3
      GO TO 10
    5 IF( M .LE. 61) GO TO 10
      M  =  61
   10 MM2  =  M-2
      MP3  =  M+3
      DO 15 I= MM2,MP3
      DNUM(I) = 1.0
      DO 15 J=MM2,MP3
      IF(I.EQ.J) GO TO 15
      DNUM(I) = DNUM(I)*(ANG - E(J))
   15 CONTINUE
      DO 20 I = MM2,MP3
      DDEN(I)  =  1.0
      DO 20 J = MM2,MP3
      IF(I.EQ.J) GO TO 16
      D  =  E(I) - E(J)
      GO TO 17
   16 D  =  1.0
   17 DDEN(I) = D*DDEN(I)
   20 CONTINUE
      XID  =  0.0
      DO 30 I=MM2,MP3
      DD = DNUM(I)*GX(I)/DDEN(I)
   30 XID = XID + DD
      RETURN
      END
```

0.99930504173577	0.99634011677196	0.99101337147674	0.98333625388463
0.97332682778991	0.96100879965205	0.94641137485840	0.92956917213193
0.91052213707850	0.88931544599511	0.86599939815409	0.84062929625258
0.81326531512279	0.78397235894334	0.75281990726053	0.71988185017161
0.68523631305423	0.64896547125465	0.61115535517239	0.57189564620263
0.53127946401989	0.48940314570705	0.44636601725346	0.40227015796399
0.35722015833766	0.31132287199021	0.26468716220876	0.21742364374000
0.16964442042399	0.12146281929612	0.07299312178779	0.02435029266342
0.00178328072169	0.00414703326056	0.00650445796897	0.00884675982636
0.01116813946013	0.01346304789671	0.01572603047602	0.01795171577569
0.02013482315353	0.02227017380838	0.02435270256871	0.02637746971505
0.02833967261425	0.03023465707240	0.03205792835485	0.03380516183714
0.03547221325688	0.03705512854024	0.03855015317861	0.03995374113272
0.04126256324262	0.04247351512365	0.04358372452932	0.04459055816375
0.04549162792741	0.04628479658131	0.04696818281621	0.04754016571483
0.04799938859645	0.04834476223480	0.04857546744150	0.04869095700913

B/A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
1	0.9466	1.0000	1.0518	1.0729	1.0780	1.0653
3	-0.0	0.0	0.0	0.0	0.0	0.0089
5	0.0359	-0.0	-0.0590	-0.0953	-0.1065	-0.0863
7	0.0	0.0	-0.0	-0.0	-0.0	-0.0155
9	0.0087	-0.0	0.0088	0.0317	0.0432	0.0289
11	0.0	0.0	0.0	0.0	0.0	0.0120
13	0.0034	-0.0	-0.0021	-0.0141	-0.0240	-0.0116
15	0.0	-0.0	-0.0	-0.0	-0.0	-0.0092
17	0.0017	-0.0	0.0005	0.0071	0.0154	0.0047
19	0.0	0.0	0.0	0.0	0.0	0.0066
21	0.0010	-0.0	-0.0002	-0.0039	-0.0107	-0.0012
23	0.0	0.0	0.0	-0.0	-0.0	-0.0046
25	0.0006	-0.0	0.0	0.0023	0.0078	-0.0003
27	0.0	-0.0	0.0	0.0	0.0	0.0030
29	0.0004	-0.0	-0.0	-0.0014	-0.0059	0.0010
31	0.0	0.0	0.0	0.0	-0.0	-0.0018
33	0.0003	-0.0	-0.0	0.0008	0.0046	-0.0012
35	0.0	0.0	0.0	0.0	0.0	0.0010
37	0.0002	-0.0	-0.0	-0.0005	-0.0036	0.0011
39	0.0	-0.0	-0.0	-0.0	-0.0	-0.0004
41	0.0002	-0.0	-0.0	0.0003	0.0029	-0.0009
43	0.0	0.0	0.0	0.0	0.0	0.0001
45	0.0001	-0.0	-0.0	-0.0002	-0.0024	0.0007
47	0.0	-0.0	-0.0	-0.0	-0.0	0.0001
49	0.0001	0.0	-0.0	0.0001	0.0020	-0.0005
51	0.0	0.0	0.0	0.0	0.0	-0.0002
53	0.0001	-0.0	-0.0	-0.0001	-0.0017	0.0003
55	0.0	0.0	0.0	0.0	-0.0	0.0003
57	0.0001	-0.0	-0.0	0.0001	0.0014	-0.0002
59	0.0	0.0	-0.0	0.0	0.0	-0.0002
61	0.0001	-0.0	-0.0	-0.0	-0.0012	0.0001
63	0.0	0.0	0.0	0.0	-0.0	0.0002
65	0.0	-0.0	-0.0	0.0	0.0010	0.0
67	0.0	0.0	0.0	0.0	0.0	-0.0002
69	0.0	-0.0	-0.0	-0.0	-0.0009	-0.0001
71	0.0	0.0	-0.0	-0.0	-0.0	0.0001
73	0.0	-0.0	-0.0	0.0	0.0007	0.0001
75	0.0	0.0	0.0	0.0	0.0	-0.0001
77	0.0	-0.0	-0.0	-0.0	-0.0006	-0.0001
79	0.0	0.0	0.0	0.0	-0.0	0.0
81	0.0	0.0	0.0	0.0	0.0006	0.0001
83	0.0	0.0	-0.0	0.0	0.0	-0.0
85	0.0	-0.0	-0.0	-0.0	-0.0005	-0.0001
87	0.0	0.0	0.0	0.0	-0.0	-0.0
89	0.0	-0.0	-0.0	0.0	0.0004	0.0
91	0.0	-0.0	-0.0	0.0	0.0	0.0
93	0.0	0.0	-0.0	-0.0	-0.0004	-0.0
95	0.0	0.0	0.0	0.0	-0.0	-0.0
97	0.0	-0.0	-0.0	0.0	0.0003	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0

B/A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
101	0.0	-0.0	-0.0	-0.0	-0.0003	-0.0
103	0.0	-0.0	-0.0	-0.0	-0.0	-0.0
105	0.0	-0.0	-0.0	0.0	0.0003	-0.0
107	0.0	0.0	0.0	0.0	0.0	0.0
109	0.0	-0.0	-0.0	-0.0	-0.0002	0.0
111	0.0	0.0	0.0	0.0	-0.0	-0.0
113	0.0	0.0	-0.0	0.0	0.0002	-0.0
115	0.0	-0.0	-0.0	-0.0	0.0	0.0
117	0.0	-0.0	-0.0	-0.0	-0.0002	0.0
119	0.0	0.0	0.0	0.0	-0.0	-0.0
121	0.0	-0.0	-0.0	-0.0	0.0002	-0.0
123	0.0	0.0	0.0	0.0	0.0	0.0
125	0.0	0.0	0.0	0.0	-0.0001	0.0
127	0.0	0.0	0.0	0.0	-0.0	0.0
129	0.0	-0.0	-0.0	-0.0	0.0001	-0.0
131	0.0	0.0	0.0	0.0	0.0	-0.0
133	0.0	0.0	0.0	0.0	-0.0001	0.0
135	0.0	-0.0	-0.0	-0.0	-0.0	0.0
137	0.0	-0.0	-0.0	-0.0	0.0001	-0.0
139	0.0	0.0	0.0	0.0	0.0	-0.0
141	0.0	-0.0	-0.0	-0.0	-0.0001	-0.0
143	0.0	0.0	0.0	0.0	-0.0	0.0
145	0.0	-0.0	-0.0	-0.0	0.0001	0.0
147	0.0	-0.0	-0.0	-0.0	0.0	-0.0
149	0.0	-0.0	-0.0	-0.0	-0.0001	-0.0
151	0.0	0.0	0.0	0.0	0.0	0.0
153	0.0	-0.0	-0.0	-0.0	0.0001	-0.0
155	0.0	0.0	0.0	0.0	0.0	-0.0
157	0.0	-0.0	-0.0	-0.0	-0.0001	-0.0
159	0.0	-0.0	-0.0	-0.0	-0.0	-0.0

B/A	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
1	0.5848	0.6061	0.6242	0.6305	0.6317	0.6244
3	0.1284	0.1534	0.1819	0.1948	0.1979	0.1866
5	0.0663	0.0743	0.0898	0.1007	0.1043	0.0983
7	0.0396	0.0441	0.0511	0.0574	0.0603	0.0585
9	0.0278	0.0290	0.0305	0.0323	0.0338	0.0358
11	0.0201	0.0203	0.0183	0.0164	0.0162	0.0209
13	0.0157	0.0148	0.0106	0.0060	0.0043	0.0108
15	0.0124	0.0111	0.0057	-0.0007	-0.0036	0.0037
17	0.0102	0.0086	0.0026	-0.0047	-0.0083	-0.0011
19	0.0085	0.0067	0.0006	-0.0068	-0.0107	-0.0043
21	0.0072	0.0053	-0.0007	-0.0076	-0.0113	-0.0062
23	0.0062	0.0043	-0.0014	-0.0073	-0.0104	-0.0071

B/A	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
25	0.0054	0.0035	-0.0018	-0.0065	-0.0086	-0.0072
27	0.0047	0.0029	-0.0019	-0.0052	-0.0063	-0.0068
29	0.0042	0.0024	-0.0018	-0.0038	-0.0038	-0.0059
31	0.0037	0.0020	-0.0017	-0.0024	-0.0014	-0.0048
33	0.0033	0.0016	-0.0015	-0.0012	0.0007	-0.0036
35	0.0030	0.0014	-0.0013	-0.0001	0.0023	-0.0023
37	0.0027	0.0012	-0.0011	0.0007	0.0034	-0.0011
39	0.0025	0.0010	-0.0008	0.0013	0.0040	-0.0001
41	0.0023	0.0008	-0.0007	0.0016	0.0040	0.0008
43	0.0021	0.0007	-0.0005	0.0017	0.0036	0.0015
45	0.0019	0.0006	-0.0004	0.0017	0.0029	0.0020
47	0.0017	0.0005	-0.0002	0.0015	0.0019	0.0022
49	0.0016	0.0005	-0.0001	0.0012	0.0009	0.0023
51	0.0015	0.0004	-0.0001	0.0008	-0.0	0.0022
53	0.0014	0.0003	-0.0	0.0005	-0.0009	0.0019
55	0.0013	0.0003	0.0	0.0002	-0.0015	0.0016
57	0.0012	0.0003	0.0	-0.0001	-0.0019	0.0011
59	0.0011	0.0002	0.0001	-0.0003	-0.0021	0.0007
61	0.0011	0.0002	0.0001	-0.0005	-0.0020	0.0003
63	0.0010	0.0002	0.0001	-0.0005	-0.0017	-0.0001
65	0.0009	0.0001	0.0001	-0.0006	-0.0012	-0.0005
67	0.0009	0.0001	0.0001	-0.0005	-0.0007	-0.0008
69	0.0008	0.0001	0.0	-0.0004	-0.0001	-0.0009
71	0.0008	0.0001	0.0	-0.0003	0.0004	-0.0010
73	0.0007	0.0001	0.0	-0.0002	0.0008	-0.0011
75	0.0007	0.0001	0.0	-0.0001	0.0011	-0.0010
77	0.0007	0.0001	0.0	0.0	0.0012	-0.0009
79	0.0006	0.0001	0.0	0.0001	0.0012	-0.0007
81	0.0006	0.0001	0.0	0.0002	0.0011	-0.0005
83	0.0006	0.0	-0.0	0.0002	0.0008	-0.0003
85	0.0006	0.0	-0.0	0.0002	0.0005	-0.0001
87	0.0005	0.0	-0.0	0.0002	0.0002	0.0001
89	0.0005	0.0	-0.0	0.0002	-0.0001	0.0003
91	0.0005	0.0	-0.0	0.0002	-0.0004	0.0004
93	0.0005	0.0	-0.0	0.0001	-0.0006	0.0005
95	0.0004	0.0	-0.0	0.0001	-0.0008	0.0006
97	0.0004	0.0	-0.0	0.0	-0.0008	0.0006
99	0.0004	0.0	-0.0	-0.0	-0.0007	0.0005
101	0.0004	0.0	-0.0	-0.0001	-0.0006	0.0004
103	0.0004	0.0	-0.0	-0.0001	-0.0004	0.0003
105	0.0004	0.0	-0.0	-0.0001	-0.0002	0.0002
107	0.0003	0.0	-0.0	-0.0001	0.0	0.0001
109	0.0003	0.0	-0.0	-0.0001	0.0002	-0.0
111	0.0003	0.0	-0.0	-0.0001	0.0004	-0.0001
113	0.0003	0.0	-0.0	-0.0001	0.0005	-0.0002
115	0.0003	0.0	-0.0	-0.0	0.0005	-0.0003
117	0.0003	0.0	-0.0	-0.0	0.0005	-0.0003
119	0.0003	0.0	-0.0	0.0	0.0005	-0.0003
121	0.0003	0.0	-0.0	0.0	0.0003	-0.0003
123	0.0003	0.0	-0.0	0.0	0.0002	-0.0003

B/A	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
125	0.0002	0.0	-0.0	0.0	0.0001	-0.0003
127	0.0002	0.0	-0.0	0.0	-0.0001	-0.0002
129	0.0002	0.0	-0.0	0.0	-0.0002	-0.0001
131	0.0002	0.0	-0.0	0.0	-0.0003	-0.0
133	0.0002	0.0	0.0	0.0	-0.0004	0.0
135	0.0002	0.0	-0.0	0.0	-0.0004	0.0001
137	0.0002	0.0	-0.0	0.0	-0.0003	0.0001
139	0.0002	0.0	-0.0	0.0	-0.0003	0.0002
141	0.0002	-0.0	-0.0	-0.0	-0.0002	0.0002
143	0.0002	0.0	-0.0	-0.0	-0.0001	0.0002
145	0.0002	-0.0	-0.0	-0.0	0.0	0.0002
147	0.0002	0.0	-0.0	-0.0	0.0001	0.0002
149	0.0002	-0.0	-0.0	-0.0	0.0002	0.0001
151	0.0002	0.0	-0.0	-0.0	0.0003	0.0001
153	0.0002	-0.0	-0.0	-0.0	0.0003	0.0001
155	0.0002	0.0	-0.0	-0.0	0.0003	0.0
157	0.0001	-0.0	-0.0	-0.0	0.0002	-0.0
159	0.0001	0.0	-0.0	-0.0	0.0002	-0.0001
161	0.0001	-0.0	-0.0	0.0	0.0001	-0.0001
163	0.0001	0.0	-0.0	0.0	0.0	-0.0001
165	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0001
167	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0001
169	0.0001	-0.0	-0.0	0.0	-0.0002	-0.0001
171	0.0001	-0.0	-0.0	0.0	-0.0002	-0.0001
173	0.0001	-0.0	-0.0	0.0	-0.0002	-0.0001
175	0.0001	0.0	-0.0	0.0	-0.0002	-0.0001
177	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0
179	0.0001	-0.0	-0.0	0.0	-0.0001	0.0
181	0.0001	-0.0	-0.0	0.0	-0.0	0.0
183	0.0001	-0.0	-0.0	-0.0	0.0	0.0001
185	0.0001	-0.0	-0.0	-0.0	0.0001	0.0001
187	0.0001	-0.0	-0.0	-0.0	0.0001	0.0001
189	0.0001	-0.0	-0.0	-0.0	0.0002	0.0001
191	0.0001	-0.0	-0.0	-0.0	0.0002	0.0001
193	0.0001	-0.0	-0.0	-0.0	0.0001	0.0001
195	0.0001	0.0	0.0	-0.0	0.0001	0.0001
197	0.0001	-0.0	-0.0	-0.0	0.0001	0.0001
199	0.0001	-0.0	-0.0	-0.0	0.0	0.0
201	0.0001	0.0	0.0	-0.0	-0.0	0.0
203	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0
205	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0
207	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0
209	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0001
211	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0001
213	0.0001	0.0	-0.0	0.0	-0.0001	-0.0001
215	0.0001	-0.0	-0.0	0.0	-0.0001	-0.0001
217	0.0001	-0.0	-0.0	0.0	-0.0	-0.0001
219	0.0001	-0.0	-0.0	0.0	-0.0	-0.0
221	0.0001	0.0	-0.0	0.0	0.0	-0.0
223	0.0001	-0.0	-0.0	0.0	0.0001	-0.0

B/A	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
225	0.0001	-0.0	-0.0	-0.0	0.0001	-0.0
227	0.0001	-0.0	-0.0	-0.0	0.0001	0.0
229	0.0001	-0.0	-0.0	-0.0	0.0001	0.0
231	0.0001	-0.0	-0.0	-0.0	0.0001	0.0
233	0.0001	-0.0	0.0	-0.0	0.0001	0.0
235	0.0001	-0.0	-0.0	-0.0	0.0	0.0
237	0.0001	-0.0	-0.0	-0.0	0.0	0.0
239	0.0001	-0.0	0.0	-0.0	-0.0	0.0
241	0.0001	-0.0	-0.0	-0.0	-0.0	0.0
243	0.0001	-0.0	-0.0	-0.0	-0.0001	0.0
245	0.0001	-0.0	-0.0	0.0	-0.0001	0.0
247	0.0001	-0.0	-0.0	0.0	-0.0001	0.0
249	0.0	-0.0	-0.0	0.0	-0.0001	0.0
251	0.0	-0.0	0.0	0.0	-0.0001	-0.0
253	0.0	-0.0	-0.0	0.0	-0.0	-0.0
255	0.0	-0.0	0.0	0.0	-0.0	-0.0
257	0.0	-0.0	-0.0	-0.0	0.0	-0.0
259	0.0	-0.0	-0.0	0.0	0.0	-0.0
261	0.0	-0.0	-0.0	0.0	0.0	-0.0
263	0.0	0.0	0.0	0.0	0.0001	-0.0
265	0.0	-0.0	-0.0	-0.0	0.0001	-0.0
267	0.0	-0.0	-0.0	-0.0	0.0001	-0.0
269	0.0	-0.0	-0.0	-0.0	0.0	-0.0

B/A	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
1	1.1693	1.2119	1.2484	1.2609	1.2635	1.2611
3	-0.2566	-0.3067	-0.3639	-0.3896	-0.3959	-0.3892
5	0.1324	0.1484	0.1795	0.2014	0.2086	0.1989
7	-0.0790	-0.0881	-0.1022	-0.1147	-0.1205	-0.1096
9	0.0555	0.0579	0.0609	0.0645	0.0675	0.0573
11	-0.0401	-0.0405	-0.0365	-0.0327	-0.0324	-0.0247
13	0.0314	0.0296	0.0213	0.0119	0.0086	0.0046
15	-0.0247	-0.0222	-0.0114	0.0014	0.0072	0.0068
17	0.0204	0.0171	0.0051	-0.0095	-0.0167	-0.0122
19	-0.0169	-0.0134	-0.0011	0.0137	0.0215	0.0133
21	0.0144	0.0107	-0.0014	-0.0152	-0.0225	-0.0118
23	-0.0123	-0.0086	0.0028	0.0147	0.0208	0.0088
25	0.0107	0.0070	-0.0035	-0.0129	-0.0173	-0.0054
27	-0.0093	-0.0057	0.0037	0.0104	0.0127	0.0022
29	0.0083	0.0047	-0.0037	-0.0077	-0.0077	0.0004
31	-0.0073	-0.0039	0.0034	0.0049	0.0028	-0.0022
33	0.0066	0.0033	-0.0030	-0.0023	0.0014	0.0031
35	-0.0059	-0.0028	0.0026	0.0002	-0.0046	-0.0034
37	0.0054	0.0023	-0.0021	0.0015	0.0069	0.0030

B/A	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
39	-0.0049	-0.0020	0.0017	-0.0026	-0.0080	-0.0023
41	0.0045	0.0017	-0.0013	0.0032	0.0080	0.0015
43	-0.0041	-0.0014	0.0010	-0.0035	-0.0072	-0.0006
45	0.0038	0.0012	-0.0007	0.0033	0.0057	-0.0001
47	-0.0035	-0.0011	0.0005	-0.0029	-0.0039	0.0006
49	0.0032	0.0009	-0.0003	0.0023	0.0018	-0.0008
51	-0.0030	-0.0008	0.0001	-0.0016	0.0001	0.0009
53	0.0028	0.0007	-0.0	0.0009	-0.0018	-0.0008
55	-0.0026	-0.0006	-0.0	-0.0003	0.0031	0.0006
57	0.0024	0.0005	0.0001	-0.0003	-0.0039	-0.0003
59	-0.0022	-0.0004	-0.0001	0.0007	0.0042	0.0001
61	0.0021	0.0004	0.0001	-0.0010	-0.0039	0.0002
63	-0.0020	-0.0003	-0.0001	0.0011	0.0033	-0.0003
65	0.0019	0.0003	0.0001	-0.0011	-0.0024	0.0004
67	-0.0018	-0.0003	-0.0001	0.0010	0.0014	-0.0004
69	0.0017	0.0002	0.0001	-0.0009	-0.0003	0.0003
71	-0.0016	-0.0002	-0.0001	0.0007	-0.0007	-0.0003
73	0.0015	0.0002	0.0001	-0.0004	0.0016	0.0001
75	-0.0014	-0.0001	-0.0	0.0002	-0.0021	-0.0
77	0.0013	0.0001	0.0	0.0	0.0024	-0.0
79	-0.0013	-0.0001	-0.0	-0.0002	-0.0024	0.0001
81	0.0012	0.0001	0.0	0.0003	0.0022	-0.0001
83	-0.0011	-0.0001	0.0	-0.0004	-0.0017	0.0001
85	0.0011	0.0001	-0.0	0.0004	0.0011	-0.0001
87	-0.0010	-0.0001	0.0	-0.0004	-0.0004	0.0001
89	0.0010	0.0001	-0.0	0.0004	-0.0003	-0.0
91	-0.0009	-0.0001	0.0	-0.0003	0.0008	-0.0
93	0.0009	0.0	-0.0	0.0002	-0.0013	0.0001
95	-0.0009	-0.0	0.0	-0.0001	0.0015	-0.0001
97	0.0008	0.0	-0.0	0.0	-0.0016	0.0001
99	-0.0008	-0.0	0.0	0.0001	0.0015	-0.0001
101	0.0008	0.0	-0.0	-0.0001	-0.0012	0.0001
103	-0.0007	-0.0	0.0	0.0002	0.0009	-0.0
105	0.0007	0.0	-0.0	-0.0002	-0.0004	0.0
107	-0.0007	-0.0	0.0	0.0002	-0.0	-0.0
109	0.0006	0.0	-0.0	-0.0002	0.0004	-0.0
111	-0.0006	-0.0	0.0	0.0001	-0.0008	0.0
113	0.0006	0.0	-0.0	-0.0001	0.0010	-0.0
115	-0.0006	-0.0	0.0	0.0001	-0.0011	0.0
117	0.0006	0.0	-0.0	-0.0	0.0010	-0.0
119	-0.0005	-0.0	0.0	-0.0	-0.0009	0.0
121	0.0005	0.0	-0.0	0.0	0.0007	0.0
123	-0.0005	-0.0	0.0	-0.0001	-0.0004	-0.0
125	0.0005	0.0	-0.0	0.0001	0.0001	0.0
127	-0.0005	-0.0	0.0	-0.0001	0.0002	-0.0
129	0.0004	-0.0	-0.0	0.0001	-0.0005	0.0
131	-0.0004	0.0	0.0	-0.0001	0.0007	-0.0
133	0.0004	0.0	0.0	0.0001	-0.0007	0.0
135	-0.0004	0.0	0.0	-0.0	0.0008	-0.0
137	0.0004	-0.0	-0.0	0.0	-0.0007	0.0

B/A	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
139	-0.0004	0.0	0.0	-0.0	0.0006	-0.0
141	0.0004	-0.0	-0.0	-0.0	-0.0004	-0.0
143	-0.0004	0.0	0.0	0.0	0.0001	0.0
145	0.0003	-0.0	-0.0	-0.0	0.0001	-0.0
147	-0.0003	0.0	0.0	0.0	-0.0003	0.0
149	0.0003	-0.0	-0.0	-0.0	0.0004	0.0
151	-0.0003	0.0	0.0	0.0	-0.0005	-0.0
153	0.0003	-0.0	-0.0	-0.0	0.0006	0.0
155	-0.0003	0.0	0.0	0.0	-0.0005	-0.0
157	0.0003	-0.0	-0.0	-0.0	0.0004	0.0
159	-0.0003	0.0	0.0	0.0	-0.0003	-0.0
161	0.0003	-0.0	-0.0	0.0	0.0002	0.0
163	-0.0003	0.0	0.0	-0.0	0.0	-0.0
165	0.0003	-0.0	-0.0	0.0	-0.0002	0.0
167	-0.0003	0.0	0.0	-0.0	0.0003	-0.0
169	0.0002	-0.0	-0.0	0.0	-0.0004	0.0
171	-0.0002	0.0	0.0	-0.0	0.0004	-0.0
173	0.0002	-0.0	-0.0	0.0	-0.0004	0.0
175	-0.0002	0.0	0.0	-0.0	0.0004	-0.0
177	0.0002	-0.0	-0.0	0.0	-0.0003	0.0
179	-0.0002	0.0	0.0	-0.0	0.0002	-0.0
181	0.0002	-0.0	-0.0	-0.0	-0.0	0.0
183	-0.0002	0.0	0.0	0.0	-0.0001	-0.0
185	0.0002	-0.0	-0.0	-0.0	0.0002	-0.0
187	-0.0002	0.0	0.0	0.0	-0.0003	0.0
189	0.0002	-0.0	-0.0	-0.0	0.0003	-0.0
191	-0.0002	0.0	0.0	0.0	-0.0003	-0.0
193	0.0002	-0.0	-0.0	-0.0	0.0003	0.0
195	-0.0002	0.0	0.0	0.0	-0.0002	-0.0
197	0.0002	-0.0	-0.0	-0.0	0.0002	0.0
199	-0.0002	0.0	0.0	0.0	-0.0001	-0.0
201	0.0002	-0.0	-0.0	-0.0	-0.0	0.0
203	-0.0002	0.0	0.0	0.0	0.0001	-0.0
205	0.0002	-0.0	-0.0	-0.0	-0.0002	-0.0
207	-0.0001	0.0	0.0	0.0	0.0002	0.0
209	0.0001	-0.0	-0.0	0.0	-0.0002	-0.0
211	-0.0001	0.0	0.0	-0.0	0.0002	0.0
213	0.0001	-0.0	-0.0	0.0	-0.0002	-0.0
215	-0.0001	0.0	0.0	-0.0	0.0001	0.0
217	0.0001	-0.0	-0.0	-0.0	-0.0001	-0.0
219	-0.0001	0.0	0.0	0.0	0.0	0.0
221	0.0001	-0.0	-0.0	-0.0	0.0001	-0.0
223	-0.0001	0.0	0.0	0.0	-0.0001	0.0
225	0.0001	-0.0	-0.0	-0.0	0.0002	-0.0
227	-0.0001	0.0	0.0	0.0	-0.0002	0.0
229	0.0001	-0.0	-0.0	-0.0	0.0002	-0.0
231	-0.0001	0.0	0.0	0.0	-0.0002	-0.0
233	0.0001	-0.0	-0.0	-0.0	0.0001	0.0
235	-0.0001	0.0	0.0	0.0	-0.0001	-0.0
237	0.0001	-0.0	-0.0	-0.0	0.0	0.0

B/A	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
239	-0.0001	0.0	0.0	0.0	0.0	-0.0
241	0.0001	-0.0	-0.0	-0.0	-0.0001	0.0
243	-0.0001	0.0	0.0	0.0	0.0001	-0.0
245	0.0001	-0.0	0.0	0.0	-0.0001	0.0
247	-0.0001	0.0	-0.0	-0.0	0.0001	-0.0
249	0.0001	-0.0	0.0	0.0	-0.0001	0.0
251	-0.0001	0.0	0.0	0.0	0.0001	0.0
253	0.0001	-0.0	0.0	0.0	-0.0001	-0.0
255	-0.0001	0.0	-0.0	-0.0	0.0	-0.0
257	0.0001	-0.0	-0.0	-0.0	0.0	-0.0
259	-0.0001	0.0	-0.0	0.0	-0.0001	0.0
261	0.0001	-0.0	-0.0	-0.0	0.0001	-0.0
263	-0.0001	0.0	0.0	0.0	-0.0001	0.0
265	0.0001	-0.0	-0.0	-0.0	0.0001	-0.0
267	-0.0001	0.0	-0.0	0.0	-0.0001	-0.0
269	0.0001	-0.0	-0.0	-0.0	0.0001	-0.0
271	-0.0001	0.0	0.0	0.0	-0.0001	-0.0
273	0.0001	-0.0	-0.0	-0.0	0.0	0.0
275	-0.0001	0.0	0.0	0.0	-0.0	0.0
277	0.0001	-0.0	-0.0	-0.0	-0.0	0.0
279	-0.0001	-0.0	0.0	0.0	0.0001	-0.0
281	0.0001	0.0	-0.0	0.0	-0.0001	0.0
283	-0.0001	0.0	0.0	0.0	0.0001	-0.0
285	0.0001	-0.0	-0.0	-0.0	-0.0001	-0.0
287	-0.0001	0.0	0.0	0.0	0.0001	-0.0
289	0.0001	-0.0	-0.0	0.0	-0.0001	0.0
291	-0.0001	0.0	0.0	-0.0	0.0	-0.0
293	0.0001	-0.0	-0.0	0.0	-0.0	0.0
295	-0.0001	0.0	0.0	-0.0	-0.0	-0.0
297	0.0001	-0.0	-0.0	0.0	0.0	0.0
299	-0.0001	0.0	0.0	-0.0	-0.0001	-0.0
301	0.0001	-0.0	0.0	0.0	0.0001	0.0
303	-0.0001	0.0	0.0	0.0	-0.0001	-0.0
305	0.0001	-0.0	-0.0	0.0	0.0001	0.0
307	-0.0001	0.0	0.0	0.0	-0.0001	-0.0
309	0.0001	-0.0	0.0	0.0	0.0	0.0
311	-0.0001	0.0	0.0	0.0	-0.0	0.0
313	0.0001	-0.0	0.0	0.0	-0.0	0.0
315	-0.0001	0.0	-0.0	0.0	0.0	-0.0
317	0.0001	-0.0	0.0	-0.0	-0.0	0.0
319	-0.0001	0.0	0.0	0.0	0.0001	0.0
321	0.0001	-0.0	-0.0	-0.0	-0.0001	-0.0
323	-0.0001	0.0	-0.0	0.0	0.0001	-0.0
325	0.0	-0.0	-0.0	-0.0	-0.0001	-0.0
327	-0.0	0.0	0.0	0.0	0.0	0.0

B/A	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
1	0.4076	0.4162	0.4224	0.4240	0.4242	0.4214
3	0.1131	0.1254	0.1368	0.1404	0.1410	0.1361
5	0.0632	0.0701	0.0791	0.0833	0.0841	0.0801
7	0.0420	0.0473	0.0546	0.0586	0.0596	0.0561
9	0.0313	0.0350	0.0410	0.0447	0.0458	0.0428
11	0.0245	0.0275	0.0324	0.0358	0.0370	0.0344
13	0.0200	0.0224	0.0265	0.0296	0.0308	0.0285
15	0.0168	0.0187	0.0222	0.0250	0.0261	0.0242
17	0.0145	0.0160	0.0190	0.0214	0.0225	0.0209
19	0.0126	0.0139	0.0164	0.0186	0.0197	0.0183
21	0.0112	0.0122	0.0144	0.0163	0.0173	0.0162
23	0.0100	0.0108	0.0127	0.0144	0.0153	0.0144
25	0.0090	0.0097	0.0113	0.0127	0.0136	0.0129
27	0.0082	0.0088	0.0101	0.0113	0.0121	0.0116
29	0.0075	0.0080	0.0091	0.0101	0.0108	0.0105
31	0.0069	0.0073	0.0082	0.0091	0.0096	0.0095
33	0.0064	0.0067	0.0074	0.0081	0.0086	0.0087
35	0.0060	0.0062	0.0068	0.0073	0.0077	0.0079
37	0.0056	0.0057	0.0062	0.0065	0.0068	0.0072
39	0.0052	0.0053	0.0057	0.0059	0.0061	0.0066
41	0.0049	0.0050	0.0052	0.0052	0.0054	0.0060
43	0.0047	0.0046	0.0048	0.0047	0.0047	0.0055
45	0.0044	0.0043	0.0044	0.0042	0.0042	0.0050
47	0.0042	0.0041	0.0040	0.0037	0.0036	0.0046
49	0.0040	0.0038	0.0037	0.0033	0.0031	0.0042
51	0.0038	0.0036	0.0034	0.0029	0.0027	0.0038
53	0.0036	0.0034	0.0032	0.0026	0.0023	0.0035
55	0.0034	0.0032	0.0030	0.0023	0.0019	0.0031
57	0.0033	0.0031	0.0027	0.0020	0.0015	0.0028
59	0.0032	0.0029	0.0025	0.0017	0.0012	0.0026
61	0.0030	0.0028	0.0023	0.0014	0.0009	0.0023
63	0.0029	0.0027	0.0022	0.0012	0.0006	0.0021
65	0.0028	0.0025	0.0020	0.0010	0.0003	0.0018
67	0.0027	0.0024	0.0019	0.0008	0.0001	0.0016
69	0.0026	0.0023	0.0017	0.0006	-0.0001	0.0014
71	0.0025	0.0022	0.0016	0.0005	-0.0003	0.0013
73	0.0024	0.0021	0.0015	0.0003	-0.0005	0.0011
75	0.0024	0.0020	0.0014	0.0002	-0.0006	0.0009
77	0.0023	0.0019	0.0013	0.0001	-0.0008	0.0008
79	0.0022	0.0019	0.0012	-0.0	-0.0009	0.0006
81	0.0022	0.0018	0.0011	-0.0002	-0.0010	0.0005
83	0.0021	0.0017	0.0010	-0.0002	-0.0011	0.0004
85	0.0020	0.0017	0.0009	-0.0003	-0.0012	0.0002
87	0.0020	0.0016	0.0009	-0.0004	-0.0013	0.0001
89	0.0019	0.0015	0.0008	-0.0005	-0.0013	0.0
91	0.0019	0.0015	0.0007	-0.0005	-0.0014	-0.0001
93	0.0018	0.0014	0.0007	-0.0006	-0.0014	-0.0001
95	0.0018	0.0014	0.0006	-0.0006	-0.0014	-0.0002
97	0.0017	0.0013	0.0006	-0.0007	-0.0014	-0.0003
99	0.0017	0.0013	0.0005	-0.0007	-0.0015	-0.0004

B/A	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
101	0.0017	0.0012	0.0005	-0.0007	-0.0015	-0.0004
103	0.0016	0.0012	0.0004	-0.0007	-0.0015	-0.0005
105	0.0016	0.0012	0.0004	-0.0008	-0.0014	-0.0005
107	0.0016	0.0011	0.0003	-0.0008	-0.0014	-0.0006
109	0.0015	0.0011	0.0003	-0.0008	-0.0014	-0.0006
111	0.0015	0.0011	0.0003	-0.0008	-0.0014	-0.0007
113	0.0015	0.0010	0.0002	-0.0008	-0.0013	-0.0007
115	0.0014	0.0010	0.0002	-0.0008	-0.0013	-0.0007
117	0.0014	0.0010	0.0002	-0.0008	-0.0012	-0.0008
119	0.0014	0.0009	0.0002	-0.0008	-0.0012	-0.0008
121	0.0013	0.0009	0.0001	-0.0008	-0.0011	-0.0008
123	0.0013	0.0009	0.0001	-0.0008	-0.0011	-0.0008
125	0.0013	0.0009	0.0001	-0.0008	-0.0010	-0.0008
127	0.0013	0.0008	0.0001	-0.0007	-0.0010	-0.0008
129	0.0012	0.0008	0.0001	-0.0007	-0.0009	-0.0008
131	0.0012	0.0008	0.0	-0.0007	-0.0008	-0.0008
133	0.0012	0.0008	0.0	-0.0007	-0.0008	-0.0008
135	0.0012	0.0008	0.0	-0.0007	-0.0007	-0.0008
137	0.0012	0.0007	-0.0	-0.0006	-0.0006	-0.0008
139	0.0011	0.0007	-0.0	-0.0006	-0.0006	-0.0008
141	0.0011	0.0007	-0.0	-0.0006	-0.0005	-0.0008
143	0.0011	0.0007	-0.0	-0.0006	-0.0005	-0.0008
145	0.0011	0.0007	-0.0	-0.0005	-0.0004	-0.0008
147	0.0011	0.0006	-0.0001	-0.0005	-0.0003	-0.0008
149	0.0011	0.0006	-0.0001	-0.0005	-0.0003	-0.0008
151	0.0010	0.0006	-0.0001	-0.0005	-0.0002	-0.0008
153	0.0010	0.0006	-0.0001	-0.0004	-0.0001	-0.0007
155	0.0010	0.0006	-0.0001	-0.0004	-0.0001	-0.0007
157	0.0010	0.0006	-0.0001	-0.0004	-0.0	-0.0007
159	0.0010	0.0006	-0.0001	-0.0003	0.0	-0.0007
161	0.0010	0.0005	-0.0001	-0.0003	0.0001	-0.0007
163	0.0009	0.0005	-0.0001	-0.0003	0.0001	-0.0006
165	0.0009	0.0005	-0.0001	-0.0003	0.0002	-0.0006
167	0.0009	0.0005	-0.0001	-0.0002	0.0002	-0.0006
169	0.0009	0.0005	-0.0001	-0.0002	0.0002	-0.0006
171	0.0009	0.0005	-0.0001	-0.0002	0.0003	-0.0005
173	0.0009	0.0005	-0.0001	-0.0002	0.0003	-0.0005
175	0.0009	0.0005	-0.0001	-0.0001	0.0003	-0.0005
177	0.0009	0.0004	-0.0001	-0.0001	0.0004	-0.0005
179	0.0009	0.0004	-0.0001	-0.0001	0.0004	-0.0004
181	0.0008	0.0004	-0.0001	-0.0001	0.0004	-0.0004
183	0.0008	0.0004	-0.0001	-0.0001	0.0004	-0.0004
185	0.0008	0.0004	-0.0001	-0.0	0.0005	-0.0003
187	0.0008	0.0004	-0.0001	-0.0	0.0005	-0.0003
189	0.0008	0.0004	-0.0001	-0.0	0.0005	-0.0003
191	0.0008	0.0004	-0.0001	0.0	0.0005	-0.0003
193	0.0008	0.0004	-0.0001	0.0	0.0005	-0.0002
195	0.0008	0.0004	-0.0001	0.0	0.0005	-0.0002
197	0.0008	0.0004	-0.0001	0.0001	0.0005	-0.0002
199	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0002

B/A	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
201	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0001
203	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0001
205	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0001
207	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0001
209	0.0007	0.0003	-0.0001	0.0001	0.0005	-0.0
211	0.0007	0.0003	-0.0001	0.0001	0.0004	-0.0
213	0.0007	0.0003	-0.0001	0.0001	0.0004	-0.0
215	0.0007	0.0003	-0.0001	0.0001	0.0004	0.0
217	0.0007	0.0003	-0.0001	0.0001	0.0004	0.0
219	0.0007	0.0003	-0.0001	0.0001	0.0004	0.0001
221	0.0007	0.0003	-0.0001	0.0002	0.0004	0.0001
223	0.0007	0.0003	-0.0001	0.0002	0.0003	0.0001
225	0.0006	0.0003	-0.0001	0.0002	0.0003	0.0001
227	0.0006	0.0003	-0.0001	0.0002	0.0003	0.0001
229	0.0006	0.0003	-0.0001	0.0002	0.0003	0.0001
231	0.0006	0.0002	-0.0001	0.0002	0.0002	0.0002
233	0.0006	0.0002	-0.0001	0.0002	0.0002	0.0002
235	0.0006	0.0002	-0.0001	0.0002	0.0002	0.0002
237	0.0006	0.0002	-0.0001	0.0002	0.0002	0.0002
239	0.0006	0.0002	-0.0001	0.0002	0.0001	0.0002
241	0.0006	0.0002	-0.0001	0.0002	0.0001	0.0002
243	0.0006	0.0002	-0.0001	0.0002	0.0001	0.0002
245	0.0006	0.0002	-0.0001	0.0002	0.0001	0.0002
247	0.0006	0.0002	-0.0001	0.0001	0.0	0.0002
249	0.0006	0.0002	-0.0001	0.0001	0.0	0.0002
251	0.0006	0.0002	-0.0001	0.0001	-0.0	0.0002
253	0.0006	0.0002	-0.0001	0.0001	-0.0	0.0003
255	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
257	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
259	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
261	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
263	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
265	0.0005	0.0002	-0.0001	0.0001	-0.0001	0.0003
267	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
269	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
271	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
273	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
275	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
277	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0003
279	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0002
281	0.0005	0.0002	-0.0001	0.0001	-0.0002	0.0002
283	0.0005	0.0002	-0.0	0.0001	-0.0002	0.0002
285	0.0005	0.0001	-0.0	0.0	-0.0002	0.0002
287	0.0005	0.0001	-0.0	0.0	-0.0002	0.0002
289	0.0005	0.0001	-0.0	0.0	-0.0002	0.0002
291	0.0005	0.0001	-0.0	0.0	-0.0002	0.0002
293	0.0005	0.0001	-0.0	0.0	-0.0002	0.0002
295	0.0004	0.0001	-0.0	0.0	-0.0002	0.0002
297	0.0004	0.0001	-0.0	0.0	-0.0002	0.0002
299	0.0004	0.0001	-0.0	0.0	-0.0002	0.0002

B/A	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
301	0.0004	0.0001	-0.0	0.0	-0.0002	0.0002
303	0.0004	0.0001	-0.0	0.0	-0.0002	0.0002
305	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0002
307	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
309	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
311	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
313	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
315	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
317	0.0004	0.0001	-0.0	-0.0	-0.0002	0.0001
319	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0001
321	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0001
323	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0001
325	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0001
327	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0001
329	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0
331	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0
333	0.0004	0.0001	-0.0	-0.0	-0.0001	0.0
335	0.0004	0.0001	-0.0	-0.0	-0.0	0.0
337	0.0004	0.0001	-0.0	-0.0	-0.0	0.0
339	0.0004	0.0001	-0.0	-0.0	-0.0	0.0
341	0.0004	0.0001	-0.0	-0.0	-0.0	-0.0
343	0.0004	0.0001	-0.0	-0.0	0.0	-0.0
345	0.0003	0.0001	-0.0	-0.0	0.0	-0.0
347	0.0003	0.0001	-0.0	-0.0	0.0	-0.0
349	0.0003	0.0001	-0.0	-0.0	0.0	-0.0
351	0.0003	0.0001	-0.0	-0.0	0.0	-0.0
353	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
355	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
357	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
359	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
361	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
363	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
365	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
367	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
369	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
371	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
373	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
375	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
377	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
379	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
381	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
383	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
385	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
387	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001
389	0.0003	0.0001	-0.0	-0.0	0.0001	-0.0001
391	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001
393	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001
395	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001
397	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001
399	0.0003	0.0001	0.0	-0.0	0.0001	-0.0001

B/A	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
1	1.2213	1.2480	1.2672	1.2720	1.2727	1.2724
3	-0.3377	-0.3756	-0.4100	-0.4213	-0.4231	-0.4223
5	0.1879	0.2095	0.2371	0.2498	0.2524	0.2511
7	-0.1245	-0.1412	-0.1634	-0.1757	-0.1788	-0.1769
9	0.0923	0.1045	0.1227	0.1341	0.1375	0.1352
11	-0.0718	-0.0818	-0.0970	-0.1074	-0.1109	-0.1082
13	0.0586	0.0665	0.0793	0.0888	0.0923	0.0892
15	-0.0489	-0.0556	-0.0665	-0.0749	-0.0784	-0.0750
17	0.0419	0.0474	0.0567	0.0643	0.0676	0.0640
19	-0.0364	-0.0411	-0.0491	-0.0558	-0.0589	-0.0551
21	0.0321	0.0361	0.0429	0.0488	0.0518	0.0478
23	-0.0285	-0.0320	-0.0379	-0.0431	-0.0458	-0.0417
25	0.0257	0.0287	0.0337	0.0382	0.0406	0.0365
27	-0.0232	-0.0259	-0.0302	-0.0340	-0.0362	-0.0321
29	0.0212	0.0235	0.0272	0.0303	0.0323	0.0282
31	-0.0194	-0.0215	-0.0245	-0.0271	-0.0288	-0.0248
33	0.0179	0.0197	0.0223	0.0243	0.0257	0.0218
35	-0.0166	-0.0182	-0.0203	-0.0218	-0.0229	-0.0191
37	0.0155	0.0168	0.0185	0.0195	0.0204	0.0168
39	-0.0144	-0.0156	-0.0169	-0.0175	-0.0181	-0.0147
41	0.0135	0.0146	0.0155	0.0157	0.0160	0.0128
43	-0.0127	-0.0136	-0.0143	-0.0140	-0.0141	-0.0111
45	0.0119	0.0128	0.0131	0.0125	0.0124	0.0096
47	-0.0113	-0.0120	-0.0121	-0.0112	-0.0108	-0.0083
49	0.0107	0.0113	0.0112	0.0099	0.0093	0.0071
51	-0.0101	-0.0106	-0.0103	-0.0088	-0.0079	-0.0060
53	0.0096	0.0100	0.0095	0.0077	0.0067	0.0051
55	-0.0091	-0.0095	-0.0088	-0.0067	-0.0055	-0.0042
57	0.0087	0.0090	0.0082	0.0059	0.0044	0.0035
59	-0.0083	-0.0086	-0.0076	-0.0051	-0.0035	-0.0028
61	0.0080	0.0081	0.0070	0.0043	0.0026	0.0022
63	-0.0076	-0.0078	-0.0065	-0.0036	-0.0017	-0.0017
65	0.0073	0.0074	0.0060	0.0030	0.0010	0.0012
67	-0.0070	-0.0071	-0.0056	-0.0024	-0.0003	-0.0008
69	0.0067	0.0067	0.0052	0.0019	-0.0004	0.0005
71	-0.0065	-0.0065	-0.0048	-0.0014	0.0009	-0.0002
73	0.0062	0.0062	0.0045	0.0009	-0.0015	-0.0001
75	-0.0060	-0.0059	-0.0041	-0.0005	0.0019	0.0003
77	0.0058	0.0057	0.0038	0.0002	-0.0024	-0.0005
79	-0.0056	-0.0055	-0.0035	0.0002	0.0027	0.0006
81	0.0054	0.0052	0.0033	-0.0005	-0.0031	-0.0008
83	-0.0052	-0.0050	-0.0030	0.0008	0.0034	0.0009
85	0.0051	0.0049	0.0028	-0.0010	-0.0036	-0.0009
87	-0.0049	-0.0047	-0.0026	0.0012	0.0038	0.0010
89	0.0047	0.0045	0.0024	-0.0014	-0.0040	-0.0010
91	-0.0046	-0.0043	-0.0022	0.0016	0.0041	0.0010
93	0.0045	0.0042	0.0020	-0.0018	-0.0042	-0.0010
95	-0.0043	-0.0040	-0.0018	0.0019	0.0043	0.0010
97	0.0042	0.0039	0.0017	-0.0020	-0.0044	-0.0010
99	-0.0041	-0.0038	-0.0015	0.0021	0.0044	0.0010

B/A	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
101	0.0040	0.0037	0.0014	-0.0022	-0.0044	-0.0009
103	-0.0039	-0.0035	-0.0013	0.0023	0.0044	0.0009
105	0.0038	0.0034	0.0012	-0.0023	-0.0043	-0.0008
107	-0.0037	-0.0033	-0.0010	0.0024	0.0043	0.0008
109	0.0036	0.0032	0.0009	-0.0024	-0.0042	-0.0007
111	-0.0035	-0.0031	-0.0008	0.0024	0.0041	0.0007
113	0.0034	0.0030	0.0007	-0.0024	-0.0040	-0.0006
115	-0.0033	-0.0029	-0.0007	0.0024	0.0039	0.0006
117	0.0032	0.0028	0.0006	-0.0024	-0.0037	-0.0005
119	-0.0032	-0.0028	-0.0005	0.0024	0.0036	0.0004
121	0.0031	0.0027	0.0004	-0.0023	-0.0034	-0.0004
123	-0.0030	-0.0026	-0.0004	0.0023	0.0033	0.0003
125	0.0029	0.0025	0.0003	-0.0023	-0.0031	-0.0003
127	-0.0029	-0.0025	-0.0002	0.0022	0.0029	0.0002
129	0.0028	0.0024	0.0002	-0.0022	-0.0027	-0.0002
131	-0.0028	-0.0023	-0.0001	0.0021	0.0025	0.0001
133	0.0027	0.0023	0.0001	-0.0020	-0.0023	-0.0001
135	-0.0026	-0.0022	-0.0	0.0020	0.0021	0.0
137	0.0026	0.0021	0.0	-0.0019	-0.0019	-0.0
139	-0.0025	-0.0021	0.0	0.0018	0.0017	-0.0
141	0.0025	0.0020	-0.0001	-0.0018	-0.0015	0.0001
143	-0.0024	-0.0020	0.0001	0.0017	0.0013	-0.0001
145	0.0024	0.0019	-0.0001	-0.0016	-0.0011	0.0001
147	-0.0023	-0.0019	0.0002	0.0015	0.0009	-0.0001
149	0.0023	0.0018	-0.0002	-0.0014	-0.0008	0.0002
151	-0.0022	-0.0018	0.0002	0.0014	0.0006	-0.0002
153	0.0022	0.0017	-0.0002	-0.0013	-0.0004	0.0002
155	-0.0022	-0.0017	0.0003	0.0012	0.0002	-0.0002
157	0.0021	0.0017	-0.0003	-0.0011	-0.0001	0.0002
159	-0.0021	-0.0016	0.0003	0.0010	-0.0001	-0.0002
161	0.0020	0.0016	-0.0003	-0.0010	0.0002	0.0002
163	-0.0020	-0.0015	0.0003	0.0009	-0.0004	-0.0002
165	0.0020	0.0015	-0.0003	-0.0008	0.0005	0.0002
167	-0.0019	-0.0015	0.0003	0.0007	-0.0006	-0.0002
169	0.0019	0.0014	-0.0003	-0.0007	0.0008	0.0002
171	-0.0019	-0.0014	0.0004	0.0006	-0.0009	-0.0002
173	0.0018	0.0014	-0.0004	-0.0005	0.0010	0.0002
175	-0.0018	-0.0013	0.0004	0.0004	-0.0011	-0.0002
177	0.0018	0.0013	-0.0004	-0.0004	0.0011	0.0002
179	-0.0018	-0.0013	0.0004	0.0003	-0.0012	-0.0002
181	0.0017	0.0013	-0.0004	-0.0002	0.0013	0.0002
183	-0.0017	-0.0012	0.0004	0.0002	-0.0013	-0.0002
185	0.0017	0.0012	-0.0004	-0.0001	0.0014	0.0002
187	-0.0016	-0.0012	0.0004	0.0001	-0.0014	-0.0002
189	0.0016	0.0012	-0.0004	-0.0	0.0015	0.0002
191	-0.0016	-0.0011	0.0004	-0.0	-0.0015	-0.0002
193	0.0016	0.0011	-0.0004	0.0001	0.0015	0.0002
195	-0.0015	-0.0011	0.0004	-0.0001	-0.0015	-0.0001
197	0.0015	0.0011	-0.0004	0.0002	0.0015	0.0001
199	-0.0015	-0.0010	0.0004	-0.0002	-0.0015	-0.0001

B/A	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
201	0.0015	0.0010	-0.0004	0.0002	0.0015	0.0001
203	-0.0015	-0.0010	0.0004	-0.0003	-0.0015	-0.0001
205	0.0014	0.0010	-0.0004	0.0003	0.0014	0.0001
207	-0.0014	-0.0010	0.0004	-0.0003	-0.0014	-0.0001
209	0.0014	0.0009	-0.0004	0.0003	0.0014	0.0001
211	-0.0014	-0.0009	0.0004	-0.0004	-0.0013	-0.0001
213	0.0014	0.0009	-0.0004	0.0004	0.0013	0.0001
215	-0.0013	-0.0009	0.0004	-0.0004	-0.0012	-0.0001
217	0.0013	0.0009	-0.0004	0.0004	0.0012	0.0001
219	-0.0013	-0.0009	0.0003	-0.0004	-0.0011	-0.0001
221	0.0013	0.0008	-0.0003	0.0005	0.0010	0.0
223	-0.0013	-0.0008	0.0003	-0.0005	-0.0010	-0.0
225	0.0012	0.0008	-0.0003	0.0005	0.0009	0.0
227	-0.0012	-0.0008	0.0003	-0.0005	-0.0008	-0.0
229	0.0012	0.0008	-0.0003	0.0005	0.0008	0.0
231	-0.0012	-0.0008	0.0003	-0.0005	-0.0007	-0.0
233	0.0012	0.0007	-0.0003	0.0005	0.0006	0.0
235	-0.0012	-0.0007	0.0003	-0.0005	-0.0005	-0.0
237	0.0011	0.0007	-0.0003	0.0005	0.0005	0.0
239	-0.0011	-0.0007	0.0003	-0.0005	-0.0004	-0.0
241	0.0011	0.0007	-0.0003	0.0005	0.0003	0.0
243	-0.0011	-0.0007	0.0003	-0.0005	-0.0002	-0.0
245	0.0011	0.0007	-0.0003	0.0004	0.0002	0.0
247	-0.0011	-0.0007	0.0003	-0.0004	-0.0001	-0.0
249	0.0011	0.0006	-0.0003	0.0004	0.0	0.0
251	-0.0010	-0.0006	0.0003	-0.0004	0.0001	-0.0
253	0.0010	0.0006	-0.0002	0.0004	-0.0001	0.0
255	-0.0010	-0.0006	0.0002	-0.0004	0.0002	-0.0
257	0.0010	0.0006	-0.0002	0.0004	-0.0003	0.0
259	-0.0010	-0.0006	0.0002	-0.0004	0.0003	-0.0
261	0.0010	0.0006	-0.0002	0.0003	-0.0004	0.0
263	-0.0010	-0.0006	0.0002	-0.0003	0.0004	-0.0
265	0.0010	0.0006	-0.0002	0.0003	-0.0005	0.0
267	-0.0010	-0.0005	0.0002	-0.0003	0.0005	-0.0
269	0.0009	0.0005	-0.0002	0.0003	-0.0006	0.0
271	-0.0009	-0.0005	0.0002	-0.0003	0.0006	-0.0
273	0.0009	0.0005	-0.0002	0.0002	-0.0006	0.0
275	-0.0009	-0.0005	0.0002	-0.0002	0.0007	-0.0
277	0.0009	0.0005	-0.0002	0.0002	-0.0007	0.0
279	-0.0009	-0.0005	0.0002	-0.0002	0.0007	-0.0
281	0.0009	0.0005	-0.0002	0.0002	-0.0007	0.0
283	-0.0009	-0.0005	0.0002	-0.0002	0.0007	-0.0
285	0.0009	0.0005	-0.0002	0.0001	-0.0007	0.0
287	-0.0008	-0.0005	0.0001	-0.0001	0.0008	-0.0
289	0.0008	0.0005	-0.0001	0.0001	-0.0008	0.0
291	-0.0008	-0.0004	0.0001	-0.0001	0.0008	-0.0
293	0.0008	0.0004	-0.0001	0.0001	-0.0008	0.0
295	-0.0008	-0.0004	0.0001	-0.0001	0.0007	-0.0
297	0.0008	0.0004	-0.0001	0.0	-0.0007	0.0
299	-0.0008	-0.0004	0.0001	-0.0	0.0007	-0.0

B/A	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
ALPHA	1.6000	2.0000	3.0000	5.0000	10.0000	1.6000
BETA	1.6000	2.0000	3.0000	5.0000	10.0000	10.0000
N	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)	CN(N)
301	0.0008	0.0004	-0.0001	0.0	-0.0007	0.0
303	-0.0008	-0.0004	0.0001	0.0	0.0007	-0.0
305	0.0008	0.0004	-0.0001	-0.0	-0.0007	0.0
307	-0.0008	-0.0004	0.0001	0.0	0.0006	-0.0
309	0.0008	0.0004	-0.0001	-0.0	-0.0006	0.0
311	-0.0007	-0.0004	0.0001	0.0001	0.0006	-0.0
313	0.0007	0.0004	-0.0001	-0.0001	-0.0005	0.0
315	-0.0007	-0.0004	0.0001	0.0001	0.0005	-0.0
317	0.0007	0.0004	-0.0001	-0.0001	-0.0005	0.0
319	-0.0007	-0.0004	0.0001	0.0001	0.0004	-0.0
321	0.0007	0.0003	-0.0001	-0.0001	-0.0004	0.0
323	-0.0007	-0.0003	0.0001	0.0001	0.0004	-0.0
325	0.0007	0.0003	-0.0001	-0.0001	-0.0003	0.0
327	-0.0007	-0.0003	0.0001	0.0001	0.0003	-0.0
329	0.0007	0.0003	-0.0001	-0.0001	-0.0003	0.0
331	-0.0007	-0.0003	0.0001	0.0001	0.0002	-0.0
333	0.0007	0.0003	-0.0001	-0.0001	-0.0002	0.0
335	-0.0007	-0.0003	0.0001	0.0001	0.0001	-0.0
337	0.0007	0.0003	-0.0	-0.0001	-0.0001	0.0
339	-0.0006	-0.0003	0.0	0.0001	0.0001	-0.0
341	0.0006	0.0003	-0.0	-0.0001	-0.0	0.0
343	-0.0006	-0.0003	0.0	0.0001	-0.0	-0.0
345	0.0006	0.0003	-0.0	-0.0002	0.0001	0.0
347	-0.0006	-0.0003	0.0	0.0001	-0.0001	-0.0
349	0.0006	0.0003	-0.0	-0.0002	0.0001	0.0
351	-0.0006	-0.0003	0.0	0.0002	-0.0002	-0.0
353	0.0006	0.0003	-0.0	-0.0001	0.0002	0.0
355	-0.0006	-0.0003	0.0	0.0001	-0.0002	-0.0
357	0.0006	0.0003	-0.0	-0.0001	0.0003	0.0
359	-0.0006	-0.0003	0.0	0.0001	-0.0003	-0.0
361	0.0006	0.0003	-0.0	-0.0001	0.0003	0.0
363	-0.0006	-0.0003	0.0	0.0001	-0.0003	-0.0
365	0.0006	0.0003	-0.0	-0.0001	0.0003	0.0
367	-0.0006	-0.0002	0.0	0.0001	-0.0004	-0.0
369	0.0006	0.0002	-0.0	-0.0001	0.0004	0.0
371	-0.0006	-0.0002	0.0	0.0001	-0.0004	-0.0
373	0.0005	0.0002	-0.0	-0.0001	0.0004	0.0
375	-0.0005	-0.0002	0.0	0.0001	-0.0004	-0.0
377	0.0005	0.0002	-0.0	-0.0001	0.0004	0.0
379	-0.0005	-0.0002	0.0	0.0001	-0.0004	-0.0
381	0.0005	0.0002	-0.0	-0.0001	0.0004	0.0
383	-0.0005	-0.0002	0.0	0.0001	-0.0004	-0.0
385	0.0005	0.0002	-0.0	-0.0001	0.0004	0.0
387	-0.0005	-0.0002	0.0	0.0001	-0.0004	-0.0
389	0.0005	0.0002	-0.0	-0.0001	0.0004	0.0
391	-0.0005	-0.0002	-0.0	0.0001	-0.0004	-0.0
393	0.0005	0.0002	0.0	-0.0	0.0004	0.0
395	-0.0005	-0.0002	-0.0	0.0001	-0.0004	-0.0
397	0.0005	0.0002	0.0	-0.0001	0.0004	0.0
399	-0.0005	-0.0002	-0.0	0.0	-0.0004	-0.0

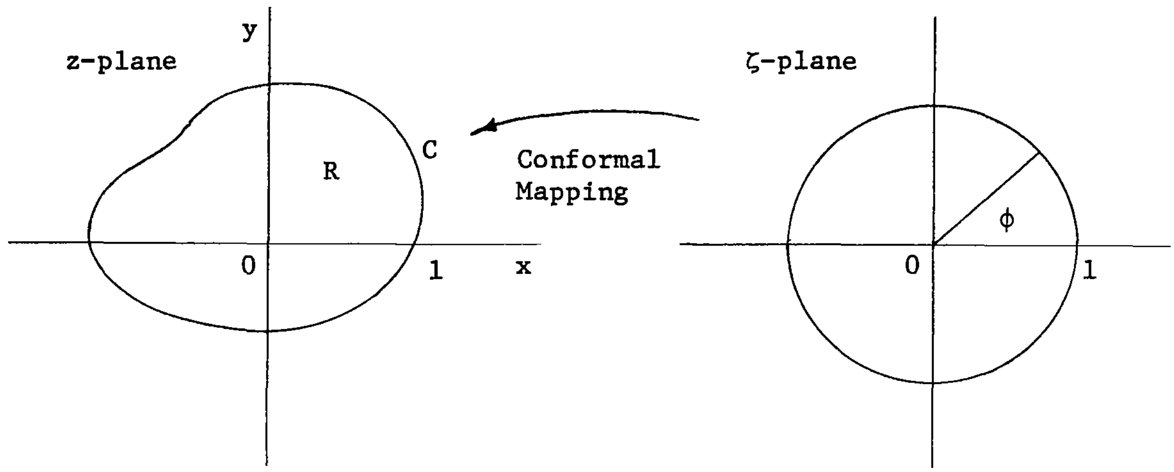

Fig. 1 Conformal Mapping of Unit Circle onto Region R

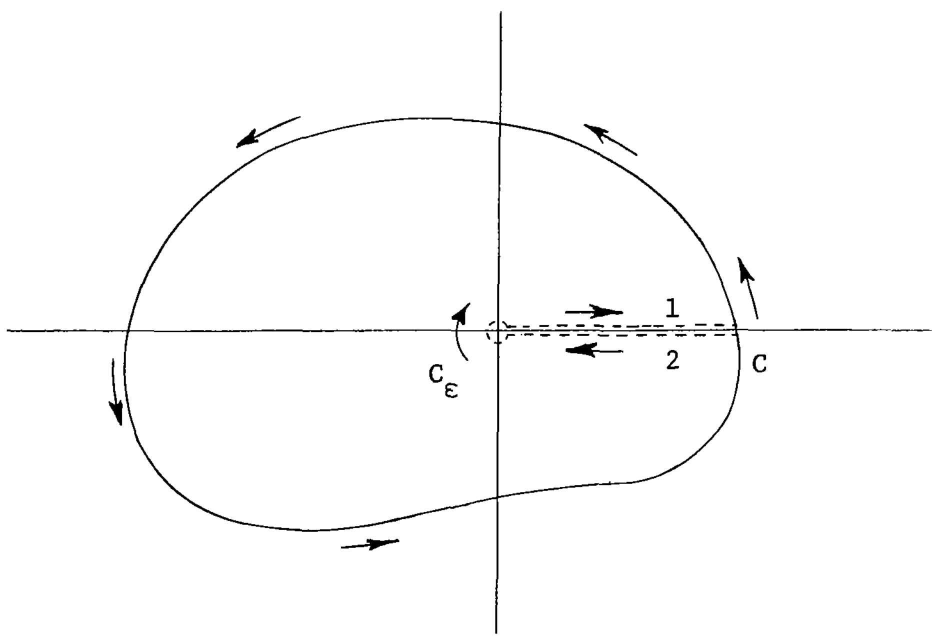

Fig. 2 Path of Integration for Equation 2

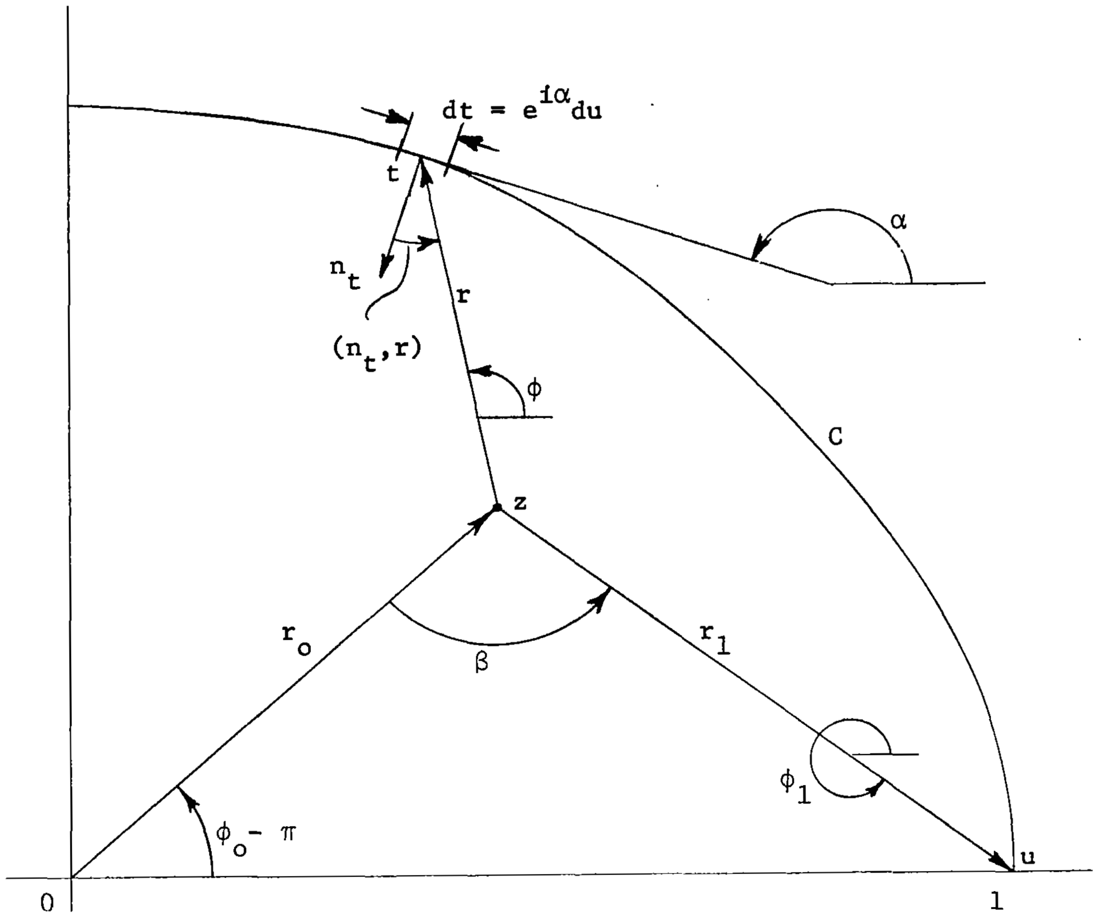

Fig. 3 Geometrical Significance of New Variables (Eq. 4)

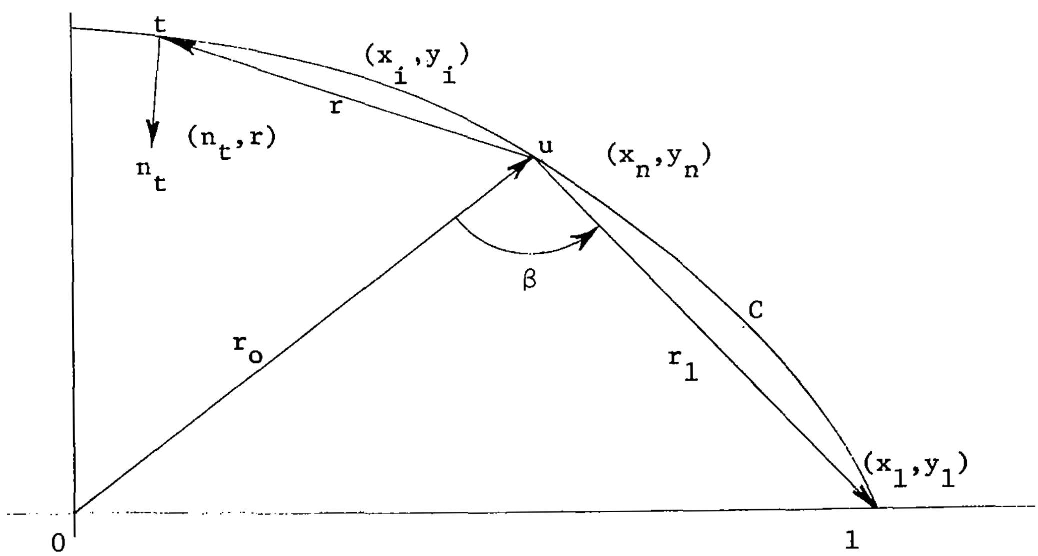

Fig. 4 Geometrical Significance of Variables when Z is on C

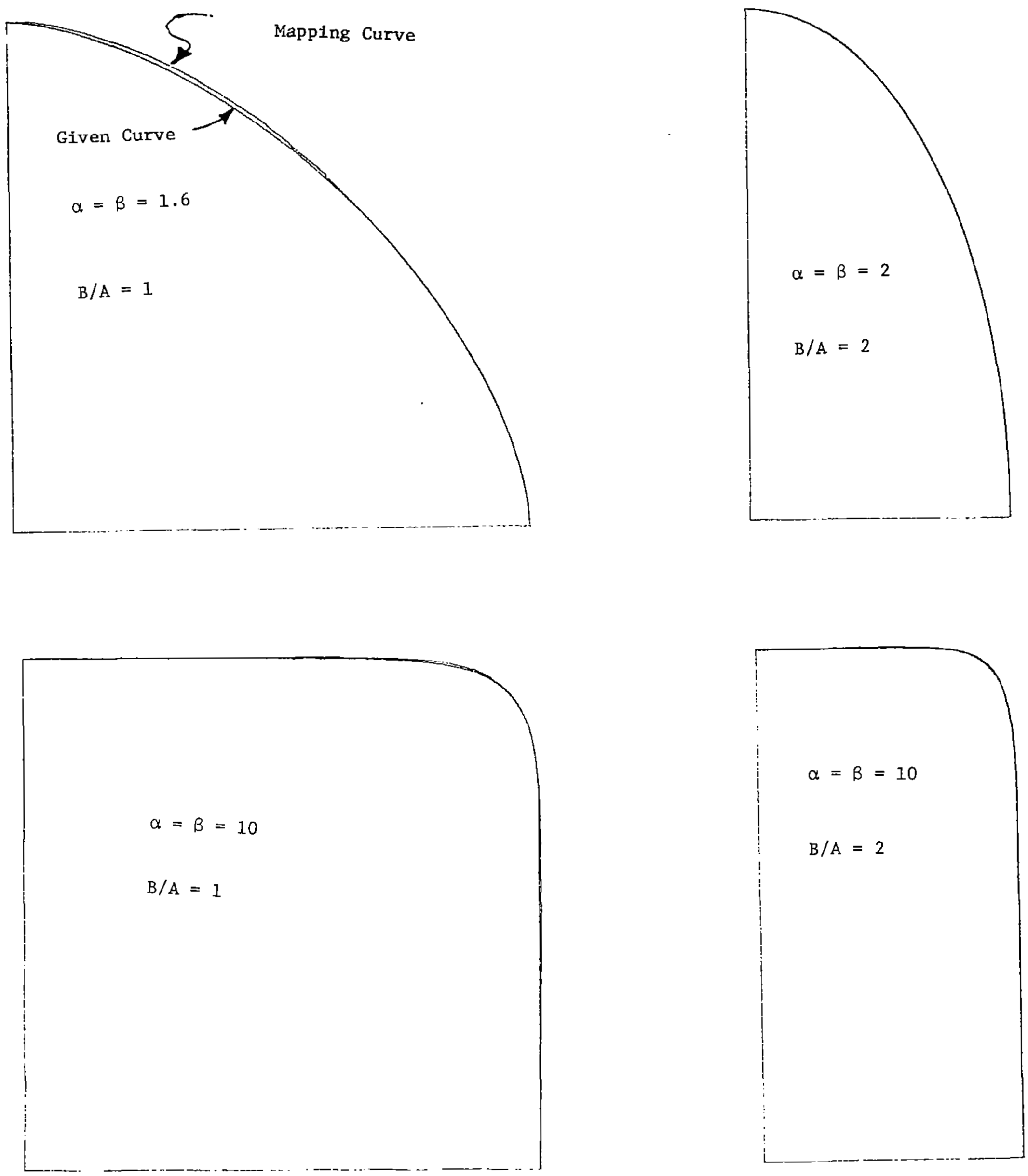

Fig. 5 Graphical Check of Accuracy of Conformal Maps, B/A = 1, 2.

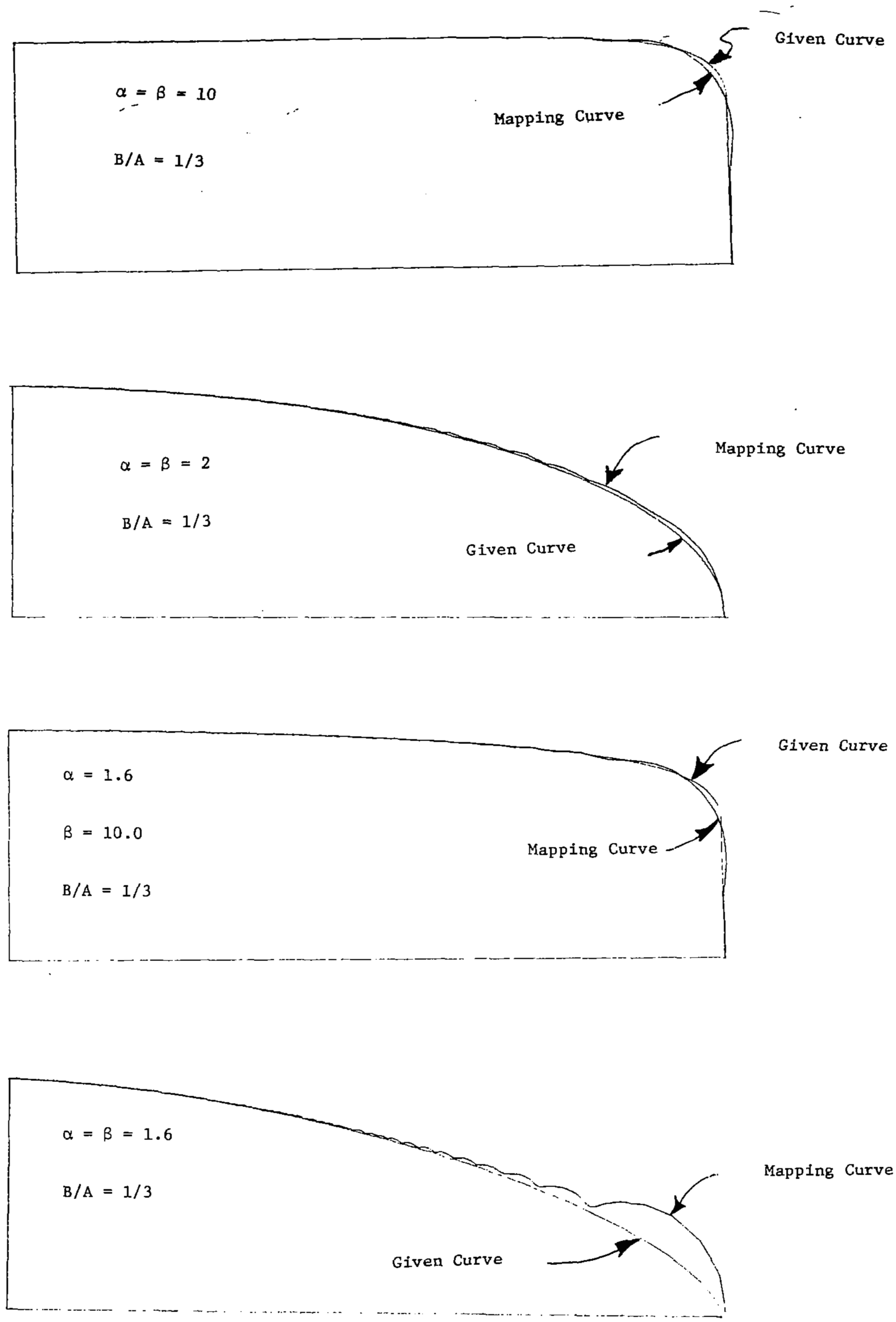

Fig. 6 Graphical Check of Accuracy of Conformal Maps, B/A = 3.